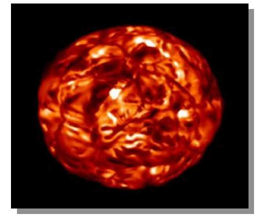
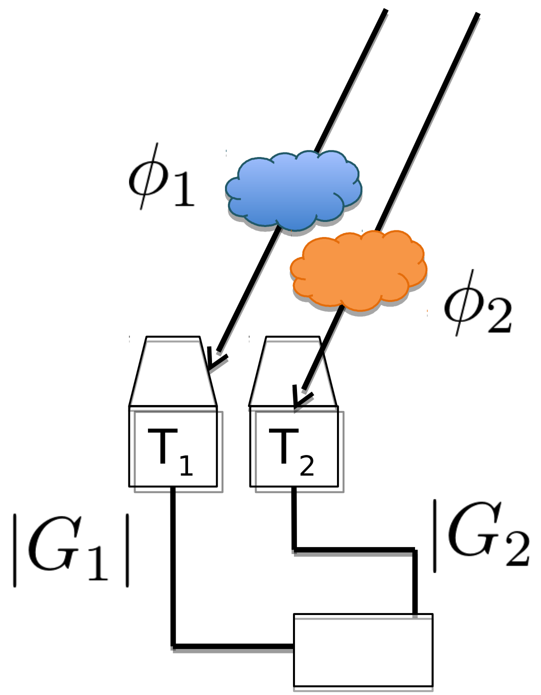
Complex closure amplitudes

Useful ?

Effects of the atmosphere and instruments

$$\mathcal{V}_{12}^{\mathrm{true}} = \left|\mathcal{V}_{12}^{\mathrm{true}}\right| e^{i\Phi_{12}^{\mathrm{true}}}$$





$$\mathcal{V}_{12}^{\mathrm{obs}} = |G_1||G_2| \left|\mathcal{V}_{12}^{\mathrm{true}}\right| e^{i\left(\Phi_{12}^{\mathrm{true}} + \phi_2 - \phi_1\right)}$$

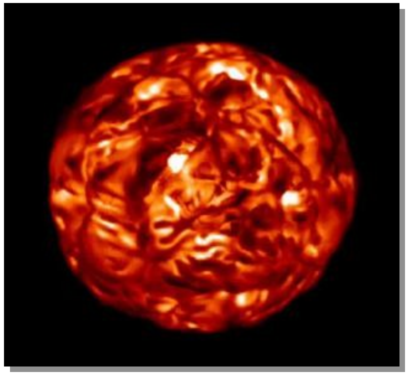
Closure phase

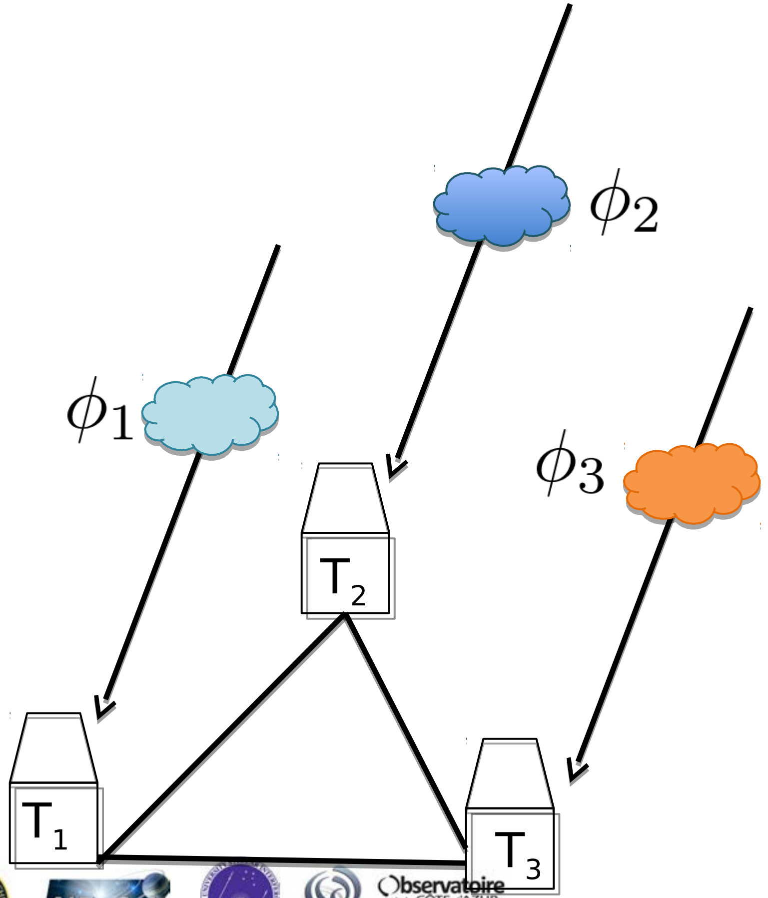
$$\Phi_{12}^{obs} = {\color{red}\Phi_{12}}^{true} + (\phi_2 - \phi_1)$$

$$\Phi_{23}^{obs} = {\color{red}\Phi_{23}}^{true} + (\phi_3 - \phi_2)$$

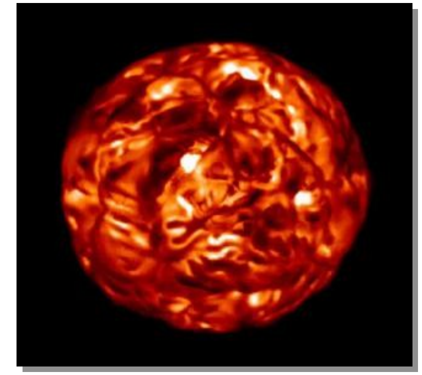
$$\Phi_{31}^{obs} = {\color{red}\Phi_{31}}^{true} + (\phi_1 - \phi_3)$$

$$CP_{123}^{obs} = \Phi_{12}^{obs} + \Phi_{23}^{obs} + \Phi_{31}^{obs}$$





Closure phase



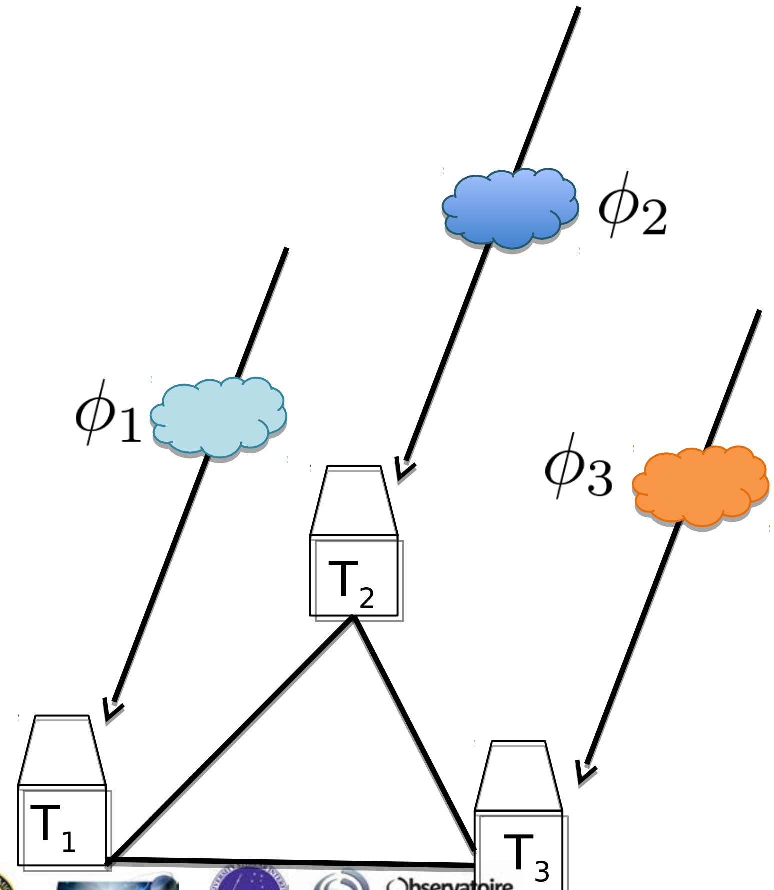
$$\Phi_{12}^{\text{obs}} = \Phi_{12}{}^{\text{true}} + (\cancel{\phi_2} - \cancel{\phi_1})$$

$$\Phi_{23}^{\text{obs}} = \Phi_{23}{}^{\text{true}} + (\cancel{\phi_3} - \cancel{\phi_2})$$

$$\Phi_{31}^{\text{obs}} = \Phi_{31}{}^{\text{true}} + (\cancel{\phi_1} - \cancel{\phi_3})$$

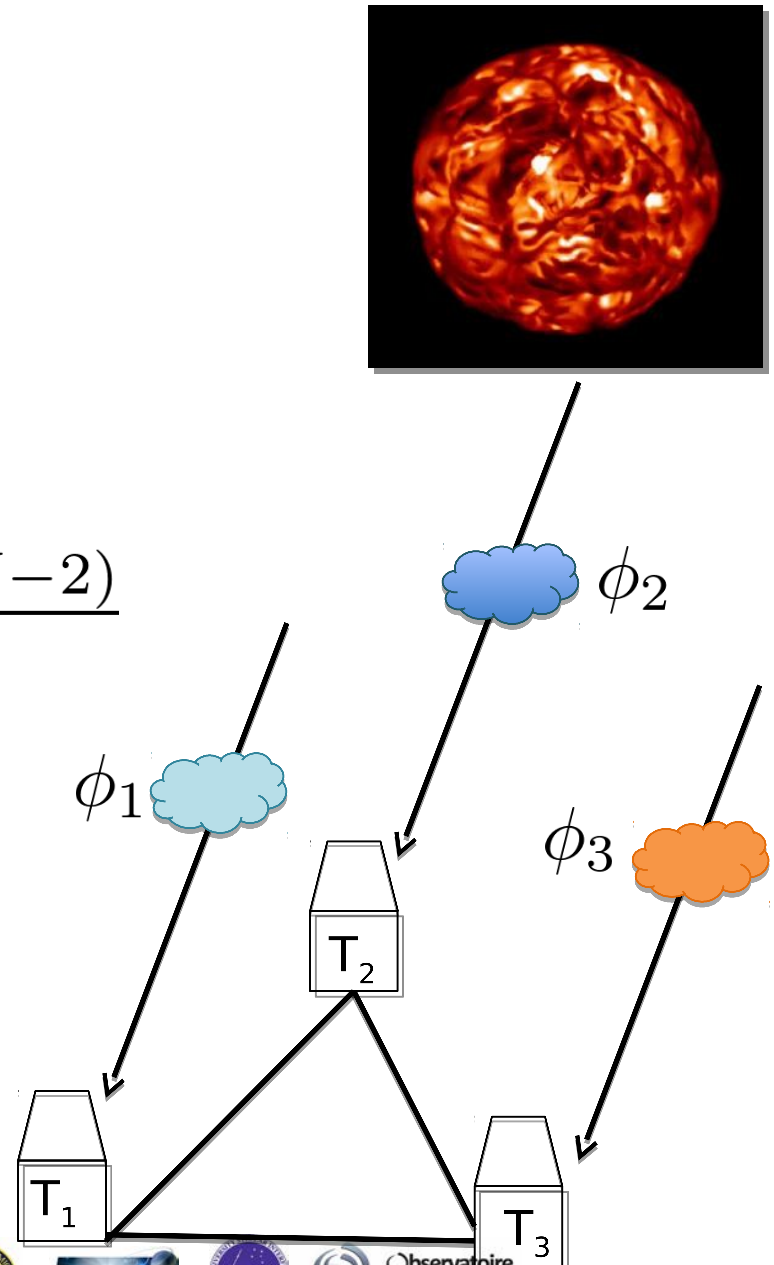
$$\text{CP}_{123}^{\text{obs}} = \Phi_{12}^{\text{obs}} + \Phi_{23}^{\text{obs}} + \Phi_{31}^{\text{obs}}$$

$$= \Phi_{12}^{\text{true}} + \Phi_{23}^{\text{true}} + \Phi_{31}^{\text{true}}$$



Closure phase

$$\mathrm{CP}_{123}^{\mathrm{obs}} = \Phi_{12}^{\mathrm{obs}} + \Phi_{23}^{\mathrm{obs}} + \Phi_{31}^{\mathrm{obs}}$$
$$= \Phi_{12}^{\mathrm{true}} + \Phi_{23}^{\mathrm{true}} + \Phi_{31}^{\mathrm{true}}$$



Independent Closure Phases $\frac{(N-1)(N-2)}{2}$

Fraction of phase information recovered $\frac{(N-2)}{N}$

3 Telescopes (CLIMB,PAVO)	33%
4 Telescopes (VLTI)	50%
6 Telescopes (MIRC)	67%
21 Telescopes (PFI)	90%

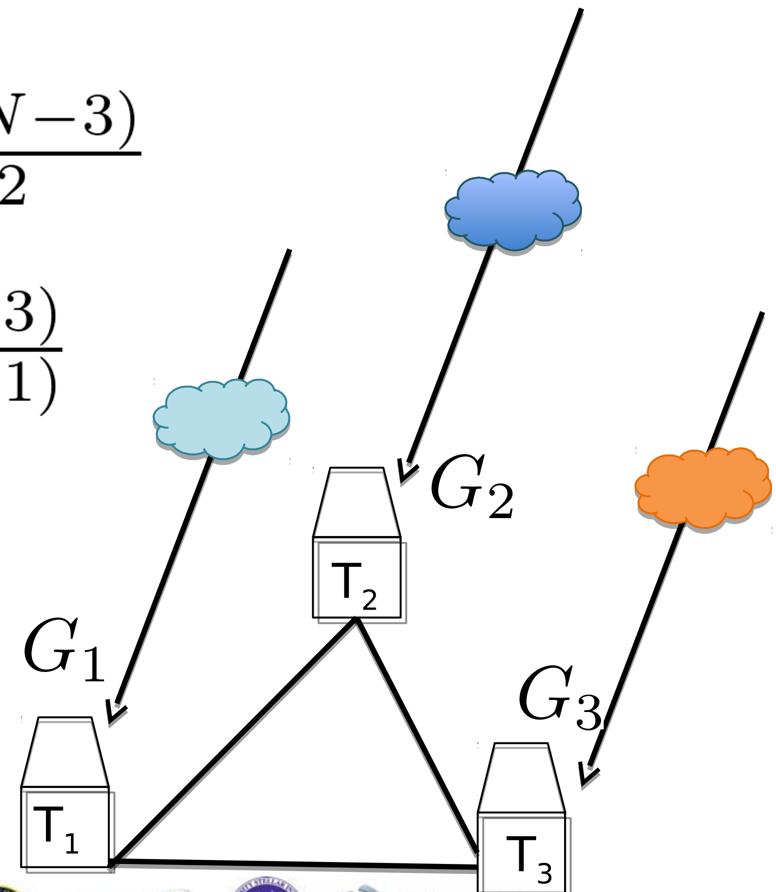
Closure amplitude

$$\mathrm{CA}_{1234} = \frac{|\mathcal{V}_{12}^{\mathrm{obs}}||\mathcal{V}_{34}^{\mathrm{obs}}|}{|\mathcal{V}_{13}^{\mathrm{obs}}||\mathcal{V}_{24}^{\mathrm{obs}}|} = \frac{|\cancel{G_1}||\cancel{G_2}||\mathcal{V}_{12}^{\mathrm{true}}||\cancel{G_3}||\cancel{G_4}||\mathcal{V}_{34}^{\mathrm{true}}|}{|\cancel{G_1}||\cancel{G_3}||\mathcal{V}_{13}^{\mathrm{true}}||\cancel{G_2}||\cancel{G_4}||\mathcal{V}_{24}^{\mathrm{true}}|}$$

Independent Closure Amplitudes $\frac{N(N-3)}{2}$

Fraction of amplitude information recovered $\frac{(N-3)}{(N-1)}$

2-3 Telescopes	none
4 Telescopes	33%
6 Telescopes	60%
21 Telescopes (PFI)	90%



Sources of amplitude variations

- Amplitude variations come from:
  - *ATMOSPHERE*
    - Fast atmosphere changes scintillation, strong in radio negligible in visible/IR
    - Slow atmosphere changes: "transfer function"
      - We use calibrators for $|\mathcal{V}|^2$
      - Closure amplitudes do not need this calibration
  - *TELESCOPES/BEAM TRAIN*
    - conventional throughput losses
    - adaptive optics
  - INSTRUMENTS
    - spatial filtering, fiber injection
  - BASELINES
    - polarization
    - ***Closure amplitudes are still affected by baseline-related decorrelation effects***

Quirks of closure amplitudes

- Loosing zeroflux value → OK: unlike radio, we did not have it anyway
- Basic bias emerging from error propagation:

$$|T4|^{\text{biased}}_{1234} = \left|\frac{\mathcal{V}_{12}\mathcal{V}_{34}}{\mathcal{V}_{14}\mathcal{V}_{23}}\right| \simeq |T4|^{\text{debiased}}_{1234}\left(1 + \frac{\sigma^2_{41}}{|\mathcal{V}^2|_{14}} + \frac{\sigma^2_{23}}{|\mathcal{V}^2|_{23}}\right)$$

Inverse SNRs

- Error on closure amplitude

$$\sigma^2_{|T4|} \simeq |T4|^2_{1234}\left(\frac{\sigma^2_{12}}{|\mathcal{V}^2|_{12}} + \frac{\sigma^2_{34}}{|\mathcal{V}^2|_{34}} + \frac{\sigma^2_{14}}{|\mathcal{V}^2|_{14}} + \frac{\sigma^2_{23}}{|\mathcal{V}^2|_{23}}\right)$$

- This is not even taking into account inherent bias from read-noise/Poisson
- Noise definitively not Gaussian-distributed
  - Division !
    - Though this is not unlike calibration with transfer function
  - Bad if denominator visibilities are low
    - Better compute inverse closure amplitude if higher SNR at numerator

Complex closure amplitude

Analogous to bispectrum T3, one can form a hereby-called T4

$$T4^{\mathrm{obs}}_{1234} = \frac{\mathcal{V}^{\mathrm{obs}}_{12}\mathcal{V}^{\mathrm{obs}}_{34}}{\mathcal{V}^{\mathrm{obs}}_{14}\mathcal{V}^{*\mathrm{obs}}_{23}} = \mathrm{CA}_{1234}\, e^{i\,\mathrm{QP}_{1234}}$$

$$T4^{\mathrm{obs}}_{1234} = \frac{|\mathcal{V}^{\mathrm{true}}_{12}||\mathcal{V}^{\mathrm{true}}_{34}|}{|\mathcal{V}^{\mathrm{true}}_{14}||\mathcal{V}^{\mathrm{true}}_{23}|}\,\frac{e^{i\left(\Phi^{\mathrm{true}}_{12}+\cancel{\phi_2}-\cancel{\phi_1}\right)}\,e^{i\left(\Phi^{\mathrm{true}}_{34}+\cancel{\phi_4}-\cancel{\phi_3}\right)}}{e^{i\left(\Phi^{\mathrm{true}}_{14}+\cancel{\phi_4}-\cancel{\phi_1}\right)}\,e^{-i\left(\Phi^{\mathrm{true}}_{23}+\cancel{\phi_3}-\cancel{\phi_2}\right)}}$$

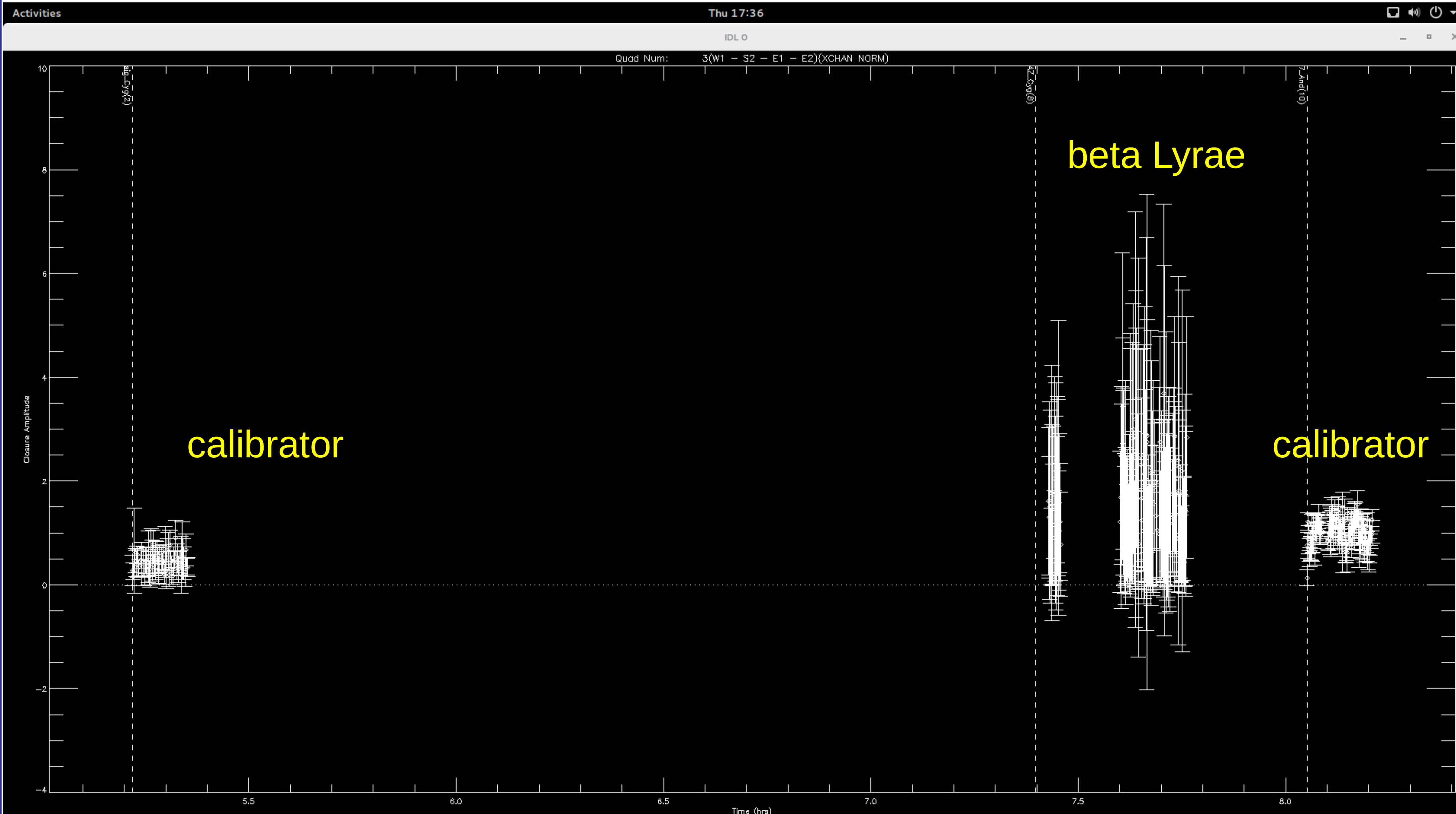
$$= T4^{\mathrm{true}}_{1234}$$

"Quad Closure Phase" $\mathrm{QP}_{1234} = \Phi^{\mathrm{true}}_{12} + \Phi^{\mathrm{true}}_{23} + \Phi^{\mathrm{true}}_{34} + \Phi^{\mathrm{true}}_{41}$

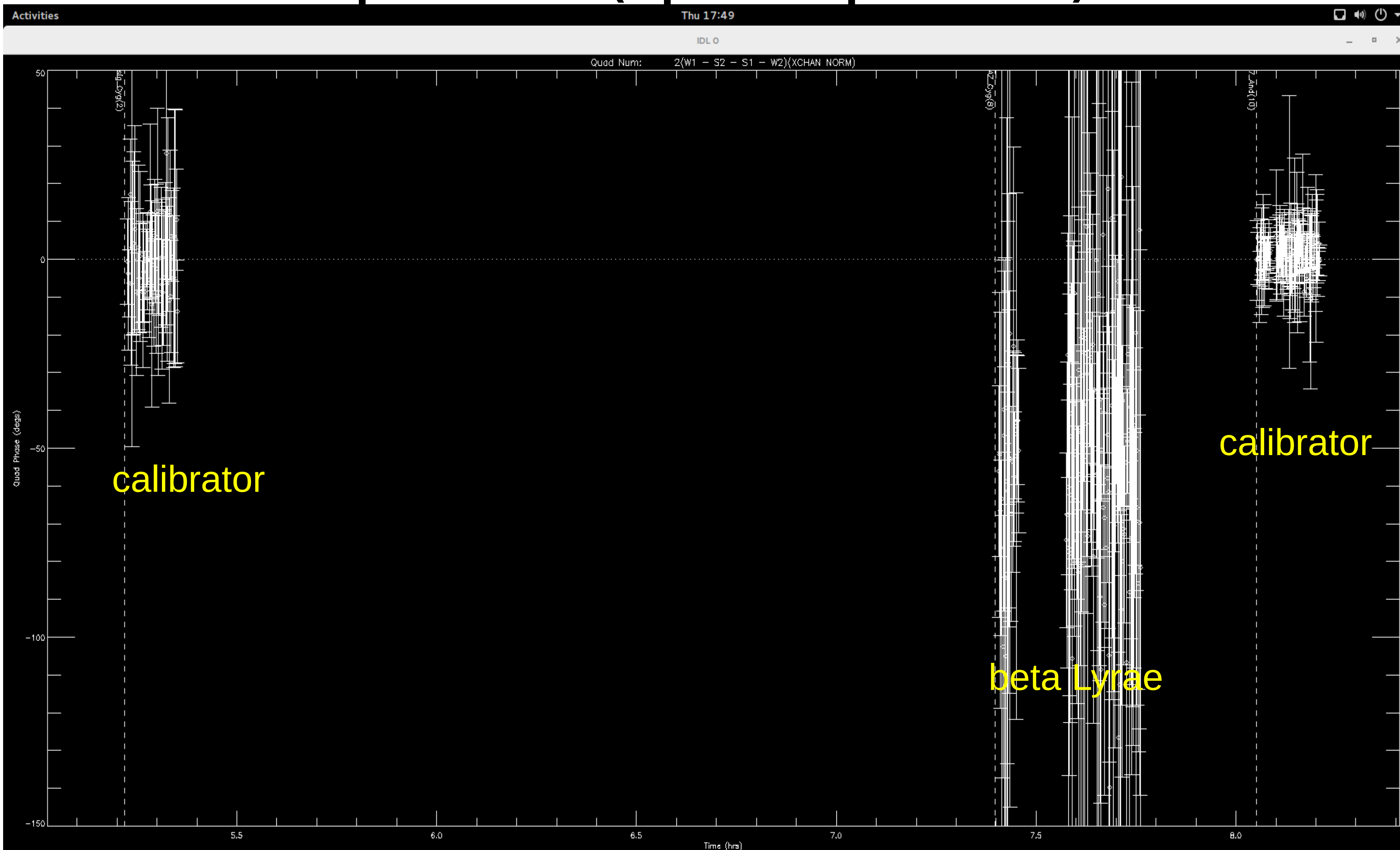
Quad closure phases

- Quad phases are part of the kernel phase
  - free observables
  - they are only ***partially*** redundant with closure phase: they have different noise statistics
    - worse SNR, being made of 4 phases instead of 3
  - number of independent quad phases is the same as the number of independent closure phases
- Like closure phases, they measure assymetric flux
- Quad phases may be more independent of flux variations than closure phases
  - Provided closure amplitudes work

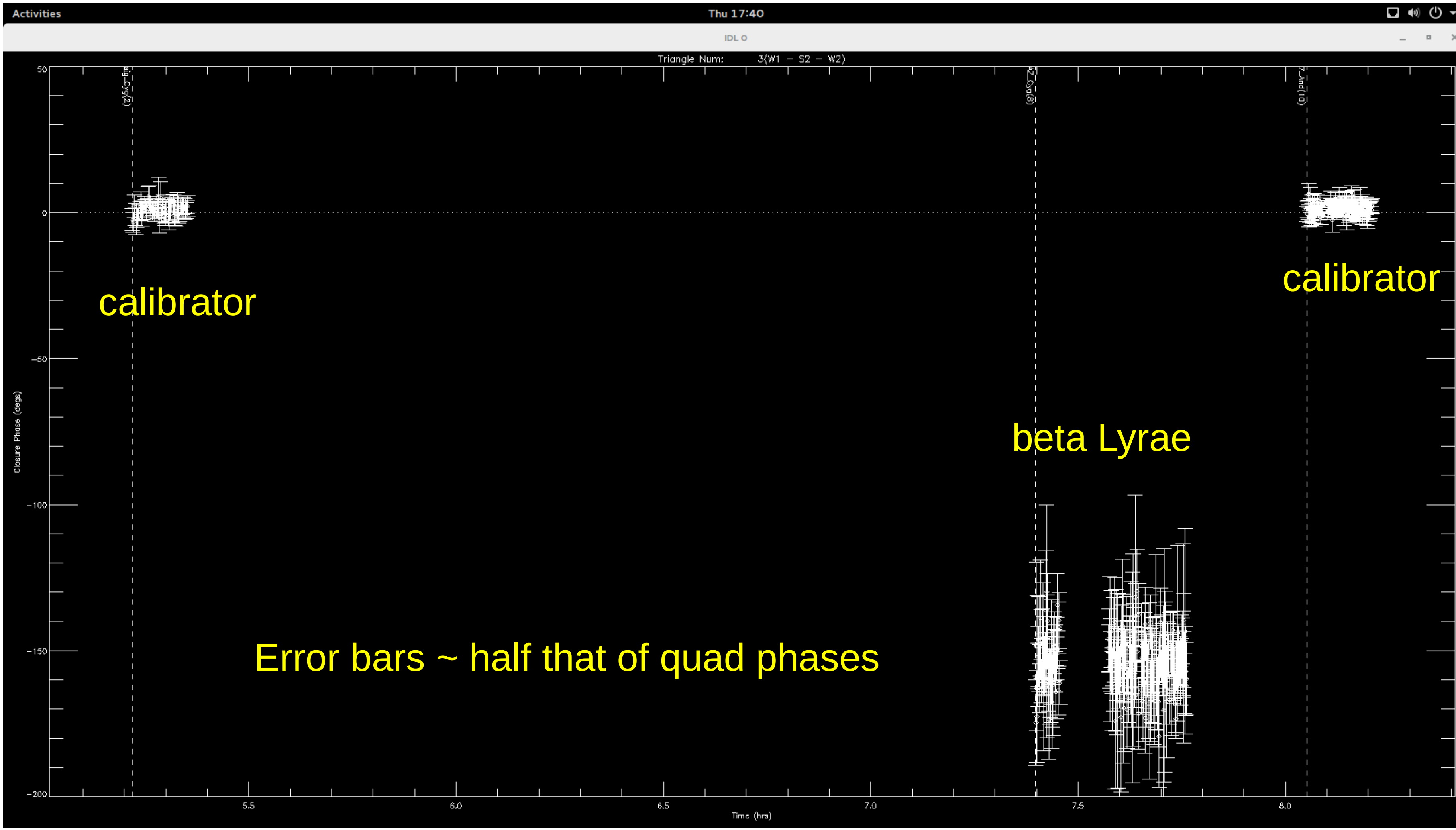
MIRC pipeline mod for T4: modulus (closure amp)



MIRC pipeline mod for T4: phase (quad phase)



Meanwhile in the closure phase world... higher SNR



Work in progress

- ✓ Ground libraries: julia and C code for handling T4 (OIFITSlib)
- ✓ Image reconstruction using T4
- ✗ In progress, simulations of T4 noise to improve debiasing from noise terms, similar to work by Gordon and Buscher (2012).
- ✗ Covariance matrix with closures phases
- ✓ First image from closure quantities only from good SNR data (but calibrated cphases)