CHARA Technical Report

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Self-weight Deflections of 8-inch Diameter Flats

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1. INTRODUCTION AND GENERAL INFORMATION

The CHARA Array will employ five 1-m size, alt-azimuth style telescopes at a site on Mount Wilson in southern California. The telescopes will be housed separately and operated remotely from a central laboratory. Light from each telescope will follow a path to the Coudé focus of the telescope, where there will be one to three reflections required to balance the polarization and phase delay properties of the various beams. The light will be directed by subsequent flat mirrors through vacuum pipes to the central laboratory. There, additional flat mirrors will further balance the polarization and phase delay, then direct the beams in the POPs, which are optical delay segments of various length, required to equalize the optical delays. The light will then be directed through a periscope arrangement into the OPLEs, for fine adjustment of optical delay. The light will pass through beam compressors, reducing the nominal beam diameter to several centimeters. Flat mirrors will then direct the beams into the beam combination room and the various parts of the beam combination system.

This report examines the self-weight deflection of 8-inch diameter optics. For the purposes of this analysis, it will be assumed that the deflections of the front surface of the mirrors should be less than 1/50 wave at $0.5\,\mu$m. This corresponds to 1/25 wave PTV error in the wavefront. This can be compared to the PTV error expected in optics as delivered from the manufacturer. For small optics, 1/25 wave PTV error on the wavefront would correspond to a mirror surface finish of about 1/50 wave RMS. Thus a 1/50 wave requirement for the mechanical support may seem a little over specified. However, figure errors due to gravitational or support deflections can combine systematically over multiple optical surfaces. This may be partially compensated by the Array's insensitivity to errors which are common to all beams. The choice of 1/50 wave PTV on the surface is thus an intuitively motivated compromise specification which appears achievable with reasonable effort and probably sufficient.

2. CONSIDERATION OF FLAT MIRRORS

The CHARA optical train may include stationary flat mirrors supported at orientation angles of 0° and 45° to the direction of gravity. The 0° orientation means that the mirror

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reflecting surface will be aligned in a vertical plane while 45° orientation means the reflecting surface will be tilted upward or downward at that angle relative to gravity. Mirror size is modest at about 8-inch diameter, but the gravity induced distortion of the mirror should be kept to a very small 1/50 wave ($\sim 0.4 \times 10^{-6}$ inches PTV).

3. FLAT MIRRORS IN A VERTICAL POSITION

A mirror in the 0° (vertical) orientation will be supported in some manner along its lower rim while being maintained in the correct vertical orientation by mechanical positioning devices. The support devices could be as simple as two "hard"" pads acting in the center-of-gravity (c.g.) plane of the mirror and spaced apart an appropriate angular distance to provide positional stability. More complex supports could involve multiple pads, slings, or even counterweighted lever supports acting around the upper rim of the mirror. However, pads are likely to be used because of their simplicity.

Gravitationally induced bending will not occur in a flat mirror in a 0° orientation. There will be a slight tendency for the mirror to droop around any discrete support points, but these will not produce angular changes in the front surface. Changes in the front surface will be due to bulging of the front surface caused by localized compression of the mirror material nearest the support pads. This effect is easy to imagine if one envisions the mirror to be made of rubber. The weight of the mirror squeezes the material nearest the support pad causing it to bulge in the orthogonal direction which pushes out the reflecting surface at that location. Poisson's Ratio for the mirror material is a direct measure of the magnitude of the bulge compared to the magnitude of the squeeze. A simplistic, but conservative, estimate of the maximum front surface bulge will be given below.

If the support pads under the mirror occupy an area of $1\,\mathrm{in}^2$ and the mirror weighs 10 lb (a reasonable estimate for an 8-inch diameter mirror, up to 1.5 inches thick made of fused silica or ULE), then the average compressive stress (S_x) is just $10\,\mathrm{lb/in}^2$. In the direction of compression, the unit deformation (ϵ_x) will be:

$$\epsilon_x = \frac{S_x}{E} \quad , \tag{1}$$

where E = Modulus of Elasticity $\gg 10 \times 10^6$ for fused silica or ULE. Unit deformation in the orthogonal direction will be:

$$\epsilon_y = \nu \frac{S_x}{E} \quad , \tag{2}$$

where ν = Poisson's Ratio $\gg 0.2$ for fused silica or ULE.

$$\epsilon_y = 0.2 \times 10^{-6} \text{ inch/inch} \tag{3}$$

If the thickness of the mirror under uniform compressive stress is 1.5 inches (i.e., the support pads extend from front to back and provide uniform contact with the glass), then the maximum deflection will be:

$$\delta_y = 1.5\ \epsilon_y \text{ inch} = 0.3 \times 10^{-6} \text{ inch} \quad . \tag{4}$$

This amounts to about 1/67th of a wave at a wavelength of 500 nm. It would appear that the compressive bulge in the front mirror surface due to support pad pressure will be small enough to neglect. The combined support pad area should be greater than $1\,\text{in}^2$ and be interfaced to the mirror in a way to distribute the load uniformly, e.g., an RTV bond or a compliant interface pad. Also, the pads should not be extended all the way to the front surface which will further reduce the bulge effect.

4. FLAT MIRRORS IN A 45° ORIENTATION

Ordinarily mirrors are supported from the rear surface, but some of the CHARA mirrors could be supported from the front near the outer rim or at the rim itself. This means that the support points are unlikely to be optimally located to minimize surface deflection due to gravity. Some of the CHARA mirrors will be mounted at a 45° angle with respect to gravity, some up-looking while others will be down-looking. One may assume that up-looking mirrors could be mounted on an optimized backside support, but the down-looking mirrors are a more difficult problem. They are most likely to be supported at three points at/near the outer rim. A 3-point support will be considered since it is a more severe case than systems with more points or a continuous rim support. A 2-point support would be the worst case, but will not be considered since it is not a stable positioning system.

Nelson et al. (1982) did an elegant study of surface deflections of circular plates with no central hole on discrete numbers of support points ranging from 1 to 36. Their objective was to determine the optimum locations for these supports when the mirror (plate) was subjected to gravity in a horizontal, up-looking orientation. Of interest here is what they were able to do with a 3-point support.

In general, Nelson et al., limited their analysis to bending effects. The effects of shear in a mirror with a small diameter-to-thickness ratio can be significant and an approximate correction factor was determined by Nelson et al. which will be applied at the end of the analysis to follow. A general expression for surface deflection (δ_{RMS}) derived by Nelson et al. follows:

$$\delta_{RMS} = \upsilon_N \left(\frac{q}{D}\right) \left(\frac{A}{N}\right)^2 \quad . \tag{5}$$

Here:

- υ_N = Support point efficiency, a term defined to measure the degree of optimization achieved by the support. For any given support arrangement, υ_N is a constant which becomes larger with as the degree of non-optimization worsens.
- q = Applied force per unit area, i.e., the unit weight of the mirror for a disc of constant thickness in a horizontal orientation with no other external forces present.
- A = Area of the mirror
- N = Number of support points
- D = Flexural rigidity of the circular plate = $E\frac{h^3}{12(1-\nu^2)}$.
- E = Modulus of Elasticity for the mirror material
- h = Mirror thickness
- ν = Poisson's Ratio for the mirror material

It is well known that the optimum arrangement for a 3-point axial support of a flat horizontal disk is for the three support points to be located on a circle with a radius equal to 0.645 times the mirror radius and, indeed, Nelson's analysis confirms this value for the optimized case. The CHARA mirrors, however, will not utilize an optimized support, but will, instead, support the mirror at or near the outer rim. The Nelson analysis also considered non-optimized supports, and provides an indication of what happens to the support efficiency when the supports are located on a ring with other than the optimum radius. For the case where $N = 3$, it is seen that the efficiency factor increases from approximately 6×10^4 at a normalized radius of 0.645 to about 25×10^4 at a normalized radius of 1.0. From this, one may infer that surface deflections of a mirror supported at three points near the rim will be slightly greater than four times the deflection of a mirror on the optimized support. This factor will be applied later.

[*** INSERT FIGURE HERE ***]

Nelson et al. calculated the support efficiency and the peak-to-valley deflections for all of the supports analyzed and for a 3-point support found the following values (these values are given in the paper):

$$\upsilon_N = 5.76 \times 10^{-3} \quad (6)$$

$$P - V \text{ deflection} = 4.2\ \delta_{RMS} \quad (7)$$

These values enable computation of the P-V deflection for one of the CHARA mirrors assuming the following conditions:

- Mirror diameter = 8 inches
- Mirror thickness = 1 inch
- Mirror density = $0.1\,\mathrm{lb/in^3}$
  (a conservative value; e.g. density of most glasses is about $0.08\,\mathrm{lb/in^3}$)
- Modulus of Elasticity = $10 \times 10^6\,\mathrm{lb/in^2}$
  (typical of fused silica or ULE. Zerodur is about 30% stiffer)
- Poisson's Ratio (ν) = 0.2
  (For fused silica and ULE; $\nu = 0.17$, for Zerodur, $\nu = 0.24$;
  the analysis by Nelson et al. assumed $\nu = 0.25$)

Substituting all of the above into the equation for δ_{RMS} yields:

$$\delta_{RMS} = 44.6 \times 10^{-9} \text{ inches} \quad (8)$$

$$\approx 1.2\,\mathrm{nm} \quad (9)$$

Applying the factors for rim-support and P-V deflection yields that the P-V deflection of a rim supported mirror (in a horizontal position) is:

$$\delta_{PTV} = (4)\,(4.2)\ \delta_{RMS} \quad (10)$$

$$= 749 \times 10^{-9} \text{ inches} \quad (11)$$

$$(\approx 19\,\mathrm{nm}, \text{ or } 1/76 \text{ wave at } 0.5\,\mu\mathrm{m.}) \quad (12)$$

Moving the mirror away from the horizontal position, the gravity component acting orthogonal to the mirror surface will be reduced by the cosine of the mounting angle of 45°. The net mirror deflection for a mirror mounted at 45° to gravity will be about

$$\cos 45^\circ \ \delta_{PTV} = 530 \times 10^{-9} \text{ inches } (13.4\,\text{nm}) \tag{13}$$

This amounts to about 1/38th of a wave at a wavelength of 500 nm. This is larger than the benchmark target of 1/50th of a wave. An improvement of about 50/38 =1.32 is required which can be achieved by increasing the mirror thickness. From the previous analysis, deflection varies inversely as the square of thickness (i.e., inversely as the cube of thickness in the D term, but q increases linearly with thickness at the same time). The required thickness will be:

$$h_{\text{req}} = h(1.32)^{1/2} = 1.15\ h \tag{14}$$

Since h was assumed to be 1.0 inch, an increase of thickness to about 1.15 inch would now be required.

The reflecting surface change computed above is due to self-weight bending of the mirror on a 3-point support. Shear effects have been neglected, but will now be estimated. The following expression, taken from Nelson et al., provides an approximate value for the total deflection including the effects due to shear:

$$\delta_{\text{total}} = \delta_{\text{bending}} \ \left[1 + \alpha \left(\frac{H}{u}\right)^2\right] \tag{15}$$

where:

- α = Constant which was determined by numerical evaluation to be ≈ 2.
- u = Distance between supports
- h = Mirror thickness (now assumed to be 1.15 inch)

Assuming an 8–inch mirror is supported by three points at the rim with equal angular separation, the value for u = 6.93 inch. From these values:

$$\begin{aligned} \delta_{\text{total}} &= 1.06\ \delta_{\text{bending}} & (16) \\ &= 562 \times 10^{-9} \text{ inches } (\approx 14.3\,\text{nm}) & (17) \end{aligned}$$

A further increase in thickness is needed to offset the 6% additional distortion due to shear. The required final thickness will be approximately (using the ratio of distortions in nm):

$$\begin{aligned} h_{\text{final}} &= \left(\frac{14.3}{13.4}\right)^{1/2} h_{\text{req}} & (18) \\ &= 1.19 \text{ inch} & (19) \end{aligned}$$

Since conservative values have been used in the calculations, an increase in mirror thickness to about 1.25 inches should be adequate to reduce front surface distortion to 1/50th wave at 500 nm wavelength.

5. REFERENCES

Nelson, J.E., Lubliner, J., and Mast, T. 1982, *SPIE*, **332**, 212.