

THE GJ 436 SYSTEM: DIRECTLY DETERMINED ASTROPHYSICAL PARAMETERS OF AN M DWARF AND IMPLICATIONS FOR THE TRANSITING HOT NEPTUNE

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ABSTRACT

The late-type dwarf GJ 436 is known to host a transiting Neptune-mass planet in a 2.6 day orbit. We present results of our interferometric measurements to directly determine the stellar diameter ($R_\star = 0.455 \pm 0.018 R_\odot$) and effective temperature ($T_{\text{EFF}} = 3416 \pm 54$ K). We combine our stellar parameters with literature time-series data, which allows us to calculate physical and orbital system parameters, including GJ 436's stellar mass ($M_\star = 0.507^{+0.071}_{-0.062} M_\odot$), stellar density ($\rho_\star = 5.37^{+0.30}_{-0.27} \rho_\odot$), planetary radius ($R_p = 0.369^{+0.015}_{-0.015} R_{\text{Jupiter}}$), and planetary mass ($M_p = 0.078^{+0.007}_{-0.008} M_{\text{Jupiter}}$), implying a mean planetary density of $\rho_p = 1.55^{+0.12}_{-0.10} \rho_{\text{Jupiter}}$. These values are generally in good agreement with previous literature estimates based on assumed stellar mass and photometric light curve fitting. Finally, we examine the expected phase curves of the hot Neptune GJ 436b, based on various assumptions concerning the efficiency of energy redistribution in the planetary atmosphere, and find that it could be constrained with *Spitzer* monitoring observations.

Key words: infrared: stars – planetary systems – stars: fundamental parameters (radii, temperatures, luminosities) – stars: individual (GJ 436) – stars: late-type – techniques: interferometric

Online-only material: color figure

1. INTRODUCTION

GJ 436 is an M3 dwarf (Kirkpatrick et al. 1991; Hawley et al. 1996) that is known to host a Neptune-sized exoplanet in a 2.64 day orbit (Cáceres et al. 2009; Ballard et al. 2010a; Southworth 2010). The planet was originally discovered by the radial velocity method (Butler et al. 2004), and subsequent photometric studies used transit photometry to determine the planet's radius and density (see Demory et al. (2007); Deming et al. (2007); Gillon et al. (2007a, 2007b); Bean et al. (2008); Pont et al. (2009); Figueira et al. (2009) and references therein). One implicit assumption in the calculation of planetary radius and density is the knowledge of the stellar radius calculated from, for instance, stellar models. Particularly in the M dwarf mass regime, however, there is a well-documented discrepancy between model radii and the ones that can be directly determined (Boyajian et al. 2010). For spectral types around M3V, the offset between model radii and directly measured counterparts is on the order of 10% (Torres 2007; López-Morales & Shaw 2007; López-Morales 2007; von Braun et al. 2008; Boyajian et al. 2010 and references therein; however, see also Demory et al. (2009).

The advent of long-baseline interferometry at wavelengths in the near-infrared or optical range has made it possible to cir-

cumvent assumptions of stellar radius by enabling direct measurements of stellar radius and other astrophysical properties for nearby, bright stars (e.g., Baines et al. 2008a, 2008b, 2009, 2010; van Belle & von Braun 2009; von Braun et al. 2011a, 2011c; Boyajian et al. 2012 and references therein). Currently the only stars known to host a transiting exoplanet with directly determined radii are HD 189733 (Baines et al. 2007) and 55 Cancri (von Braun et al. 2011b). In this paper, we use interferometric observations to obtain GJ 436's astrophysical parameters and provide a physical characterization of the system. We describe our observations in Section 2 and discuss directly determined and derived stellar and planetary astrophysical properties in Sections 3 and 4, respectively. We summarize and conclude in Section 5.

2. INTERFEROMETRIC OBSERVATIONS

Our observational methods and strategy are described in detail in von Braun et al. (2011a). We repeat aspects specific to the GJ 436 observations below.

GJ 436 was observed over four nights in 2011 January using the Georgia State University Center for High Angular Resolution Astronomy (CHARA) Array (ten Brummelaar et al. 2005), a long-baseline interferometer located at Mount Wilson Observatory in Southern California. Two of CHARA's longest baselines, S1E1 (330 m) and S1W1 (278 m), were used to

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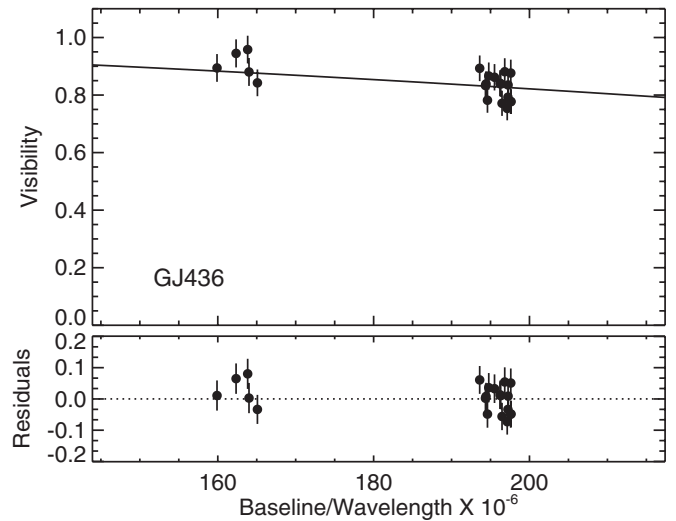
Table 1
Observation Log

UT Date	Baseline	No. of Brackets	Calibrator HD
2011 Jan 21	S1/E1	6	HD 95804, HD 102555
2011 Jan 22	S1/W1	3	HD 95804
2011 Jan 24	S1/E1	7	HD 95804, HD 102555, HD 104349
2011 Jan 25	S1/E1	3	HD 102555, HD 104349
2011 Jan 25	S1/W1	2	HD 104349

collect the observations in H band ($\lambda_{\text{central}} = 1.67 \mu\text{m}$) with the CHARA Classic beam combiner (Sturmann et al. 2003; ten Brummelaar et al. 2005) in single-baseline mode. Table 1 lists the observations, each of which contains around 2.5 minutes of integration and 1.5 minutes of telescope slewing per object (target and calibrator—see below). During our observations, GJ 436 was between 60 and 80 deg elevation.

The interferometric observations employed the common technique of taking bracketed sequences of the object and calibrator stars to remove the influence of atmospheric and instrumental systematics. We rotated between four calibrators over the observation period to minimize any systematic effects in measuring the diameter of GJ 436, whose faintness and small angular size make it a non-trivial system to observe with CHARA. The calibrator stars used in our observations were HD 95804 (spectral type A5; $\theta_{\text{EST}} = 0.211 \pm 0.009$ mas), HD 102555 (F2; $\theta_{\text{EST}} = 0.220 \pm 0.007$ mas), HD 103676 (F2; $\theta_{\text{EST}} = 0.267 \pm 0.009$ mas), and HD 104349 (K1 III; $\theta_{\text{EST}} = 0.263 \pm 0.008$ mas).¹² As in von Braun et al. (2011a, 2011b), calibrator stars were chosen to be near-point-like sources of similar brightness as GJ 436 and located at small angular distances from it.

The uniform disk and limb-darkened angular diameters (θ_{UD} and θ_{LD} , respectively; see Table 2) are found by fitting the calibrated visibility measurements (Figure 1) to the respective functions for each relation. These functions may be described as n th-order Bessel functions that are dependent on the angular diameter of the star, the projected distance between the two telescopes and the wavelength of observation (see Equations 2 and 4 of Hanbury Brown et al. 1974). Visibility is the normalized amplitude of the correlation of the light from two telescopes. It is a unitless number ranging from 0 to 1, where 0 implies no correlation, and 1 implies perfect correlation. An unresolved source would have a perfect correlation of 1.0 independent of the distance between the telescopes (baseline). A resolved

**Figure 1.** Calibrated visibility observations along with the limb-darkened angular diameter fit for GJ 436. Note the two different baseline lengths. For details, see Section 2.

object will show a decrease in visibility with increasing baseline length. The shape of the visibility versus baseline is a function of the topology of the observed object (the Fourier Transform of the object's shape). For a uniform disk this function is a Bessel function, and for this paper, we use a simple model of a limb-darkened variation of a uniform disk. The visibility of any source is reduced by a non-perfect interferometer, and the point-like calibrators are needed to calibrate out the loss of coherence caused by instrumental effects. We use the linear limb-darkening coefficient $\mu_H = 0.3688$ from the PHOENIX models in Claret (2000) for stellar $T_{\text{EFF}} = 3400$ K and $\log g = 5.0$ to convert from θ_{UD} to θ_{LD} . The uncertainties in the adopted limb-darkening coefficient amount to 0.2% when modifying the adopted gravity by 0.5 dex or the adopted T_{EFF} by 200K, well within the errors of our diameter estimate. Finally, we calculate the effect of baseline smearing due to the finite diameters of the telescope, and we find that its magnitude is around two orders of magnitude below our error estimates.

Our interferometric measurements yield the following values for GJ 436's angular diameters: $\theta_{\text{UD}} = 0.405 \pm 0.013$ mas and $\theta_{\text{LD}} = 0.417 \pm 0.013$ mas (Table 2).

3. DIRECTLY DETERMINED PARAMETERS

In this section, we present our direct measurements of GJ 436's stellar diameter, T_{EFF} , and luminosity, based on our interferometric measurements and spectral energy distribution

¹² θ_{EST} corresponds the estimated angular diameter of the calibrator stars based on spectral energy distribution fitting.

Table 2
Directly Determined and Literature Stellar Properties of GJ 436

Parameter	Value	Reference
Spectral type ...	M3V	Kirkpatrick et al. (1991); Hawley et al. (1996)
Parallax (mas) ...	98.61 ± 2.33	van Leeuwen (2007)
$V - K$...	4.513	Bessell (2000); Cutri et al. (2003)
θ_{UD} (mas) ...	0.405 ± 0.013	This work (Section 2)
θ_{LD} (mas) ...	0.417 ± 0.013	This work (Section 2)
Radius R_* (R_{\odot}) ...	0.455 ± 0.018	This work (Section 3.1)
Luminosity (L_{\odot}) ...	0.0253 ± 0.0012	This work (Section 3.2)
T_{EFF} (K) ...	3416 ± 54	This work (Section 3.2)

Note. For details, see Section 3.

(SED) fitting to literature photometry. Literature stellar and planetary astrophysical parameters for the GJ 436 system, based on spectroscopic analysis, calibrated photometric relations, and time-series photometry data, can be found in the works of, e.g., Gillon et al. (2007a), Deming et al. (2007), Maness et al. (2007), Torres (2007), Bean et al. (2008), Coughlin et al. (2008), Ballard et al. (2010b), Stevenson et al. (2010), Beaulieu et al. (2011), Knutson et al. (2011), and Southworth (2008, 2009, 2010).

3.1. Stellar Diameter from Interferometry

Based on GJ 436's limb-darkened angular diameter 0.417 ± 0.013 mas (Section 2) and trigonometric parallax from van Leeuwen (2007), we obtain a directly determined physical radius of $R_* = 0.455 \pm 0.018 R_\odot$ (see Table 2).

When comparing this value to the radius predicted by the Baraffe et al. (1998) stellar models,¹³ we reproduce the radius discrepancy for M dwarfs mentioned above and shown in Boyajian et al. (2010): our directly determined radius ($0.455 R_\odot$) exceeds the theoretical one ($0.409 R_\odot$) by 11%. This aspect is further discussed in Torres (2007), where the radius of GJ 436 is found to be overly inflated for its mass.

Otherwise indirectly calculating the stellar radius of GJ 436 requires assumption or knowledge of the stellar mass, published values of which are consistent in Maness et al. (2007) and Torres (2007). Maness et al. (2007) estimate a mass of $0.44 \pm 0.04 M_\odot$, derived from the empirically driven mass-luminosity relation in Delfosse et al. (2000). Following methods outlined in Seager & Mallén-Ornelas (2003) and Sozzetti et al. (2007), Torres (2007) calculates GJ 436's mass ($0.452^{+0.014}_{-0.012} M_\odot$) and radius ($0.464^{+0.011}_{-0.009} R_\odot$) by simultaneously applying observational constraints along with adjusting a correction factor to resolve the differences seen when comparing results to models.

Based on light curve analysis of the transiting planet and the stellar mass values from Maness et al. (2007) or Torres (2007), literature values of the stellar radius of GJ 436 reported in Gillon et al. (2007a, 2007b, $0.436 \pm 0.016 R_\odot$, $0.440 \pm 0.04 R_\odot$, respectively), Deming et al. (2007, $0.47 \pm 0.02 R_\odot$), Shporer et al. (2009, $0.45 \pm 0.02 R_\odot$), Ballard et al. (2010b, $0.437 \pm 0.016 R_\odot$), Southworth (2010, $0.454 \pm 0.029 R_\odot$),¹⁴ and Knutson et al. (2011, $0.437 \pm 0.005 R_\odot$), produce stellar radius values that are agreement with our result. Only the radius determined in Bean et al. (2008, $0.505^{+0.029}_{-0.020} R_\odot$), based on light curve analysis and the parameters in Maness et al. (2007), is slightly discrepant ($\simeq 2\sigma$). We refer the reader to the excellent Torres (2007) paper for more background and details on the assumptions and techniques employed by the studies mentioned above.

The agreements between calculated values and our interferometric radius clearly illustrate the usefulness of exoplanet light curve analysis for the determination of stellar parameters.

The question can be asked whether an interferometric measurement obtained during planetary transit or during the presence of star spots could yield a radius estimate that is thus artificially reduced. As seen in van Belle (2008), the effect on observed visibility amplitude of a transiting planet is expected to be very small ($\delta V < 0.005$ predicted for HD 189733; GJ 436 with its similar size will have similar magnitude effects). While a closure phase signal may be detectable in the near future

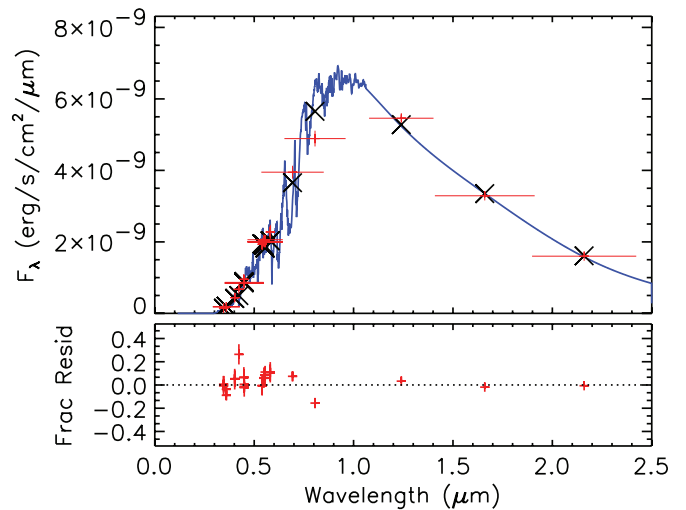


Figure 2. SED fit for GJ 436. The (blue) spectrum is an M3V spectral template (Pickles 1998). The (red) crosses indicate photometry values from the literature. “Error bars” in x-direction represent bandwidths of the filters used. The (black) X-shaped symbols show the flux value of the spectral template integrated over the filter transmission. The lower panel shows the residuals around the fit in fractional flux units of photometric uncertainty. For details, see Section 3.2.

(A color version of this figure is available in the online journal.)

($\delta CP \simeq 0.2$ deg), visibility variations in a single data point due to a transiting planet will impact current diameter measurements at the $\delta\theta \simeq 0.6\%$ level, buried in the measurement noise of the result based on all of our visibility measurements. Furthermore, planetary transits feature a contrast between the stellar surface and the obscuring or darkening feature that is significantly higher than for star spots. Thus, for spots with lower contrast ratios, expected visibility variations are even smaller.

3.2. Stellar Effective Temperature and Luminosity from SED Fitting

We produce a fit of the stellar SED based on the spectral templates of Pickles (1998) to literature photometry published in Golay (1972), Rufener (1976), Mermilliod (1986), Stauffer & Hartmann (1986), Doinidis & Beers (1991), Leggett (1992), Weis (1993), Weis (1996), and Cutri et al. (2003). In our fit, interstellar extinction is a free parameter and calculated to be $A_V = 0.000 \pm 0.014$ mag, consistent with expectations for a nearby star. The value for the distance to GJ 436 is adopted from van Leeuwen (2007). The SED fit for GJ 436, along with its residuals, is shown in Figure 2.

From the SED fit, we calculate the value of GJ 436's stellar bolometric flux to be $F_{\text{BOL}} = (7.881 \pm 0.0497) \times 10^{-9} \text{ erg cm}^{-2} \text{ s}^{-1}$, and consequently, its luminosity of $L = 0.0253 \pm 0.0012 L_\odot$. Combination with the rewritten version of the Stefan–Boltzmann Law

$$T_{\text{EFF}}(\text{K}) = 2341 (F_{\text{BOL}}/\theta_{\text{LD}}^2)^{\frac{1}{4}}, \quad (1)$$

where F_{BOL} is in units of $10^{-8} \text{ erg cm}^{-2} \text{ s}^{-1}$ and θ_{LD} is in units of mas, produces GJ 436's effective temperature to be $T_{\text{EFF}} = 3416 \pm 54 \text{ K}$ (Table 2). Due to the grazing nature of the transit in the GJ 436 system, the radius calculated for the planet is more dependent upon the limb-darkening models than the equivalent for central transits. Knowing the stellar effective temperature to a higher precision than before thus provides particularly important constraints for this system.

¹³ For this simple calculation, we assume a 5 Gyr age, solar metallicity, and a $0.44 M_\odot$ mass for GJ 436.

¹⁴ See also: <http://www.astro.keele.ac.uk/jkt/tepcat/homogeneous-par-err.html>.

Table 3
MCMC Limb-darkening Coefficients

Band	a_1	a_2	a_3	a_4
IRAC4	0.72352	-1.01134	0.79690	-0.24584
V	0.31310	1.16686	-0.85892	0.22686
R	0.37984	0.74086	-0.33350	0.018000
<i>HST</i> /NICMOS	1.533	-2.234	1.913	-0.643

Notes. Limb-darkening coefficients used in the MCMC analysis of GJ 436, based on Claret & Bloemen (2011) for IRAC4, V, and R, and from Pont et al. (2009) for *HST*/NICMOS. See Section 4.1.

Using the approach in von Braun et al. (2011a, 2011b), we calculate the system’s habitable zone to be located at 0.16–0.31 AU from GJ 436, clearly beyond the orbit of GJ 436b ($a \approx 0.03$ AU; Section 4.3 and Table 4).

4. DERIVED PARAMETERS AND RESULTS

In this section, we use our directly measured stellar parameters (Section 3 and Table 2) and combine them with a global analysis of literature time-series photometry and radial velocity (RV) data to obtain a characterization of the system as a whole, including stellar and planetary physical and orbital parameters. Rather than being required to assume a stellar mass to calculate stellar radius, we are in the position to take our measured value for R_* and calculate a stellar mass. In addition, we simulate the thermal phase curve of GJ 436b based on our calculated system parameters.

4.1. MCMC Analysis

In a transiting exoplanet system, the shape of the light curve depends in part on the mean stellar density (e.g., Seager & Mallén-Ornelas 2003; Tingley et al. 2011). Sozzetti et al. (2007) illustrated this now widely used method as an alternative to spectroscopically determined surface gravity, $\log g$, to derive more precise system parameters for the host star and its transiting planet. This technique thus enables comparing the mean stellar density and any additionally spectroscopically determined temperature and metallicity to stellar evolution models (e.g., Hebb et al. 2010; Enoch et al. 2010).

We follow the technique described in Collier Cameron et al. (2007) and combine publicly available¹⁵ RV and light curve data on GJ 436, along with data from Pont et al. (2009; F. Pont, private communication), in a global Markov Chain Monte Carlo (MCMC) analysis, thereby setting our interferometric stellar radius, effective temperature, and associated uncertainties to be fixed. Thus, we are able to make an independent measurement of the mass of this late-type star and realistic associated uncertainty.

We use fourth-order limb-darkening coefficients from Claret & Bloemen (2011) for the IRAC4 and Johnson–Cousins V and R bands, for $\log g = 4.5$ and $[\text{Fe}/\text{H}] = 0.0$ (Rojas-Ayala et al. 2010). We interpolate the values at GJ 436’s effective temperature (Section 3.2) and surface gravity (determined iteratively in the MCMC program) and perform several independent MCMC runs to explore the effect of the adopted limb-darkening coefficients on the derived stellar and planetary parameters. We test values generated from both the ATLAS and Phoenix model atmospheres and try two different values for the microturbulence (1.0 and 2.0 km s⁻¹). We find that the different limb-darkening values do not introduce any significant variations to

the derived parameters. Our results are based on ATLAS model atmospheres, $T_{\text{EFF}} = 3416$ K, $\log g$ interpolated at 4.8, and microturbulence velocity of 1.0 km s⁻¹. The limb-darkening coefficients we use in our MCMC analysis are given in Table 3.

We use the following data sets for our analysis:

1. V- and R-band transit photometry from Gillon et al. (2007a, 2007b). See Figure 3.
2. *Hubble Space Telescope* (*HST*) NICMOS transit photometry from Pont et al. (2009). See Figure 3.
3. *Spitzer*/IRAC4 8 μm transit and eclipse photometry from Deming et al. (2007), Demory et al. (2007), and Knutson et al. (2011) as binned by the authors. We adopt the Knutson et al. (2011) parameters used for the correction of their instrumental effects. See Figures 3 and 4.
4. Re-reduced and consolidated RV data presented in Maness et al. (2007) based on data from Butler et al. (2004), Butler et al. (2006), and Maness et al. (2007), with Julian Data (JD) time stamps converted to heliocentric JD to match the photometry data. See Figure 5.

Three publicly available data sets were not included in our analysis due the reasons mentioned below. The higher photometric rms in the light curves in Coughlin et al. (2008) gave this particular data set extremely little weight in the MCMC analysis. The *HST* F583W transit photometry data from Bean et al. (2008) are a combination of small 90 minute orbit segments and feature very little out-of-transit data, making it difficult to make appropriate systematic corrections to these segmented data. The *K*-band transit photometry data from Cáceres et al. (2009) have red noise at the level of 1.57 mmag and are missing the pre-transit data, including the first contact point. We note that removing these light curves did not affect the derived properties, but reduced the uncertainties. In general, the parameters changed by $< 1\sigma$, except for the impact parameter, which varied by $\sim 2\sigma$.

We characterize the system using 10 proposed parameters that form an approximately orthogonal basis set of uncorrelated parameters that fully characterize the system. Initial guesses are assigned to their values and associated uncertainties. Our set of proposed parameters are the time of minimum light, T_0 ; the orbital period, P ; the depth of the transit, δ ; the duration from first to fourth contact points, t_T ; the impact parameter, b ; the values of the eccentricity multiplied by the sine and cosine of the argument of periastron, $e \sin \omega$ and $e \cos \omega$; the semi-amplitude of the RV curve, K_1 ; the flux decrement during the secondary eclipse, Δf_2 ; and our stellar radius, R_* . The proposed stellar radius values at each step are taken from a Gaussian distribution with a mean and σ given by our measured value and uncertainty (Table 2), respectively.

At each step in the Markov chain, values for all of the proposed parameters are used to generate model light curves (Mandel & Agol 2002) and radial velocity curves. Parameter sets are either accepted or rejected based on the χ^2 value when comparing these model curves to the observed time-series data, and the accepted parameter values map out the joint posterior probability distribution. The proposed parameters are used to analytically derive the physical parameters for the system, like planet mass and radius, mean stellar density, and stellar mass. We run the routine for five independent MCMC chains. Each chain has a 2,000 step burn-in phase and completes after 10,000 accepted steps with an acceptance rate of $\sim 5\%$. Therefore, we have tested more than a million trial parameter sets to derive the resulting best-fitting parameters and their 1σ uncertainties.

¹⁵ NASA Exoplanet Archive (<http://exoplanetarchive.ipac.caltech.edu>).

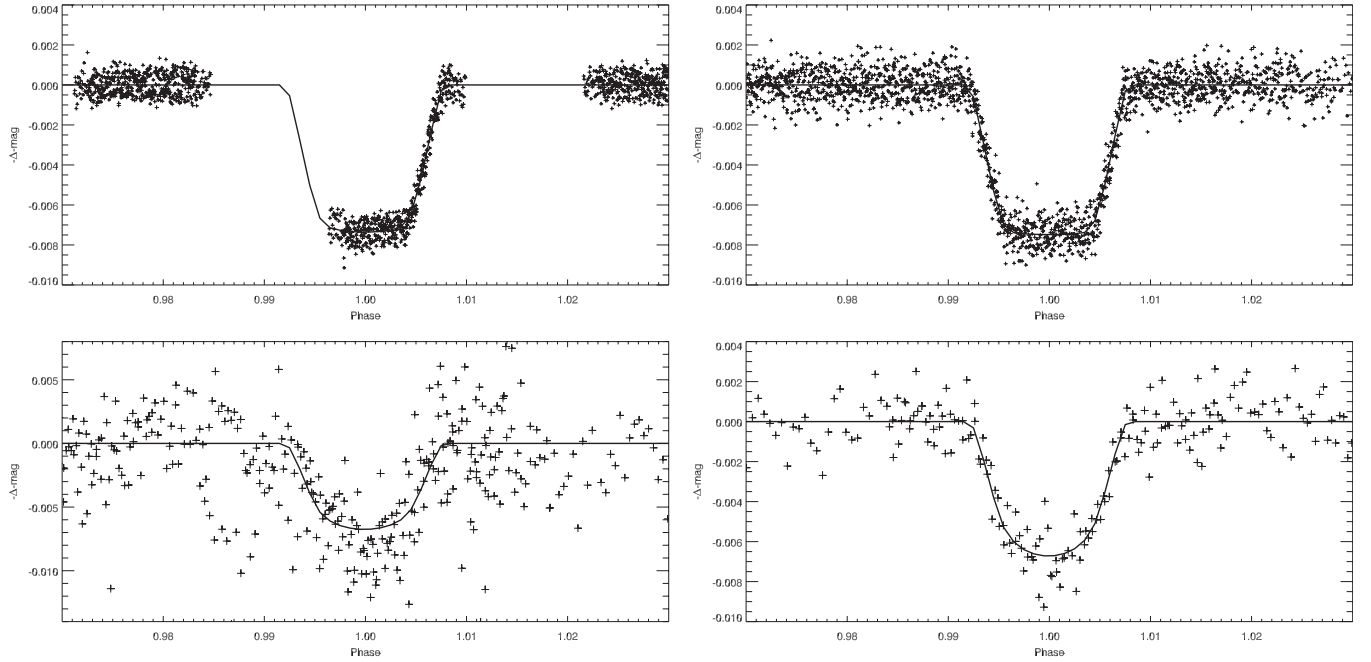


Figure 3. Literature light curves, phase folded with ephemeris information in Table 4. The respective model fit generated by our MCMC Analysis is superimposed in every panel. Top left panel: *HST* NICMOS data from Pont et al. (2009); top right: IRAC4 primary transit light curve data from Deming et al. (2007) and Knutson et al. (2011); bottom left: ground-based *R* data from Gillon et al. (2007a, 2007b); bottom right: ground-based *V* data from Gillon et al. (2007a, 2007b). For details, see Section 4.

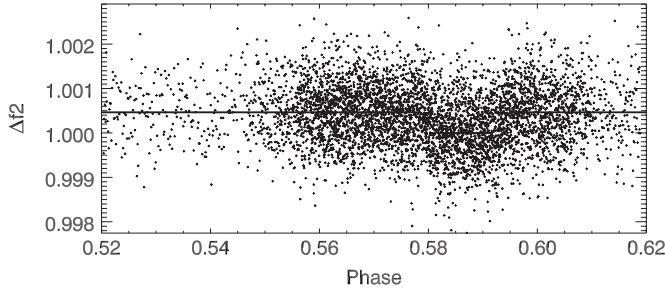


Figure 4. IRAC4 (8 μm) secondary eclipse light curve data from Demory et al. (2007) and Knutson et al. (2011), phase folded using the ephemeris information in Table 4, with our planet model overplotted. For details, see Section 4.

4.2. Derived Stellar and Planetary Parameters

The results of our MCMC analysis with the directly determined parameters and uncertainties (Table 2) as fixed input values are given in Table 4. To provide a graphical insight into the quality of our results, we show in Figures 3–5 the transit/eclipse/RV fits generated by our model, superimposed onto the literature light/RV curves described in Section 4.1. Further, we illustrate pairwise correlations between individual parameters in Figures 7 and 8.

We do not find any significant deviations in the planet parameters from what has been previously presented in the literature, which is expected given the agreement between our measured stellar radius and the calculated ones, as we discuss in Section 3.1. The two principal differences, however, between our approach and the ones listed in Section 3.1 are: (1) we incorporate RV data into our MCMC analysis, and (2) our mass value of GJ 436 ($0.507^{+0.071}_{-0.062} M_{\odot}$; Table 4) is calculated rather than assumed and thus independent of metallicity and other model assumptions.

We note that despite the volume of data used and the superb precision of especially the *Spitzer* data, the final (averaged)

Table 4
Derived GJ 436 System Parameters and 1σ Error Limits

Parameter	Symbol	Value	Units
Transit epoch (BJD) ^a ...	T_0	$2454510.80096^{+0.00005}_{-0.00005}$	days
Orbital period ^a ...	P	$2.64389826^{+0.00000056}_{-0.00000058}$	days
Transit depth ^a ...	$(R_p/R_*)^2 \equiv \delta$	$0.00694^{+0.00003}_{-0.00003}$	
Transit duration ^{a,b} ...	t_T	$0.0416^{+0.0002}_{-0.0002}$	days
Impact parameter ^a ...	b	$0.853^{+0.003}_{-0.003}$	R_*
Secondary eclipse depth ^a ...	Δf_2	$0.00046^{+0.00003}_{-0.00002}$	
Stellar reflex velocity ^a ...	K_1	$0.018^{+0.001}_{-0.001}$	km s^{-1}
Orbital semimajor axis...	a	$0.030^{+0.001}_{-0.001}$	AU
Orbital inclination...	i	$86.6^{+0.1}_{-0.1}$	degrees
Orbital eccentricity...	e	$0.146^{+0.006}_{-0.004}$	
Longitude of periastron...	ω	-21^{+5}_{-5}	degrees
eccentricity $\times \cos(\omega)$ ^a	$e \cos \omega$	$0.13653^{+0.00026}_{-0.00027}$	
eccentricity $\times \sin(\omega)$ ^a	$e \sin \omega$	$-0.05196^{+0.01396}_{-0.01534}$	
Stellar mass...	M_*	$0.507^{+0.071}_{-0.062}$	M_{\odot}
Stellar surface gravity...	$\log g_*$	$4.83^{+0.03}_{-0.03}$	(cgs)
Stellar density...	ρ_*	$5.37^{+0.30}_{-0.27}$	ρ_{\odot}
Planet radius...	R_p	$0.369^{+0.015}_{-0.015}$	R_{Jupiter}
Planet mass...	M_p	$0.078^{+0.007}_{-0.008}$	M_{Jupiter}
Planet surface gravity...	$\log g_p$	$3.12^{+0.03}_{-0.03}$	(cgs)
Planet density...	ρ_p	$1.55^{+0.12}_{-0.10}$	ρ_{Jupiter}

Notes. Derived system parameters of GJ 436 from MCMC analysis. Note that the measured system parameters are given in this table. For details, see Section 4.

^a Proposed parameters in MCMC analysis (Section 4.1), along with R_* from Table 2.

^b Defined here as the time between first and fourth contacts.

precision on the stellar mass is only at the level of about 13%. We believe this is due to the non-zero eccentricity of the system and the high impact parameter, i.e., the almost grazing transit,

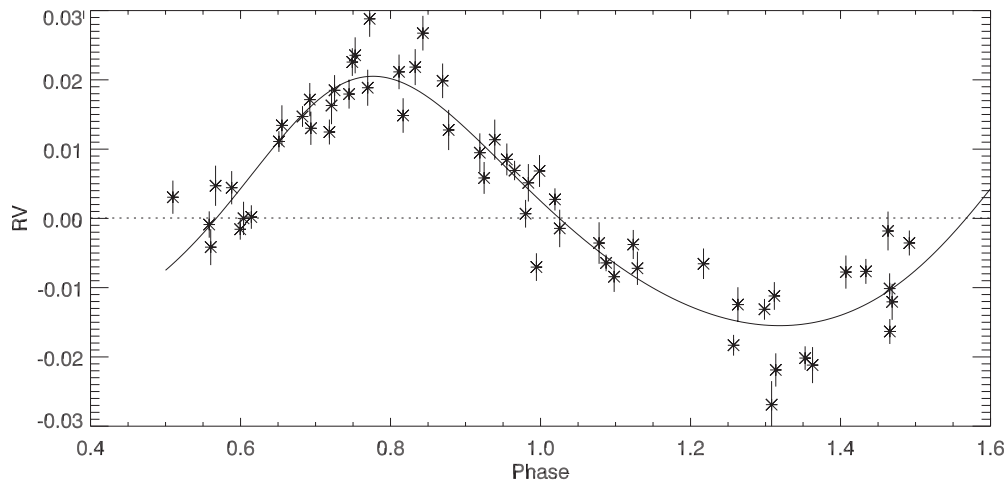


Figure 5. Radial velocity data from Maness et al. (2007), superimposed with the model fit generated by our MCMC analysis. The RV units are in km s^{-1} . For details, see Section 4.

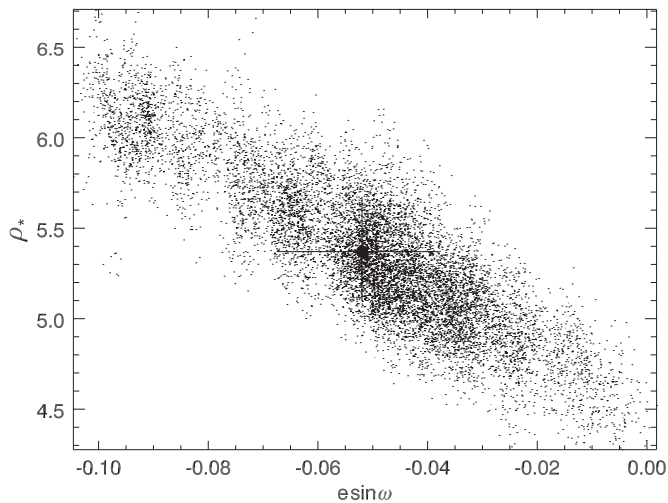


Figure 6. Correlation diagram between $e \sin \omega$ and ρ_* (in units of ρ_\odot) from our MCMC analysis, along with respective value and uncertainty from Table 4. $e \sin \omega$ has a large associated relative uncertainty that propagates to the mean stellar density and thus causes a relatively large uncertainty in the stellar mass. For details, see Section 4.2 and Table 4.

which both influence the precision of the density measurement. In particular, $e \sin \omega$ has a large associated relative uncertainty that propagates through to the stellar mass through its effect on the mean density, as shown in Figure 6. While the mid-time of the secondary eclipse provides an extremely strong constraint on $e \cos \omega$, the degeneracy between the stellar density, ρ_* , and $e \sin \omega$ means that the secondary eclipse duration provides only a weak constraint on $e \sin \omega$. Furthermore, the radial velocity data is unable to break the degeneracy between eccentricity and ω due to the small radial velocity amplitude relative to its uncertainties. In Figures 7 and 8, we show the joint posterior probability distribution of a subset of the final proposed and physical parameters, respectively.

Finally, it is worth pointing out that our value for R_p , calculated from our interferometrically determined stellar radius and the transit depth in the literature light curves (Figures 3 and 4), could be wavelength dependent due to spots on the stellar surface during transit (Ballerini et al. 2012) or attenuation of stellar radiation in the planetary atmosphere (e.g., Figure 14 in Knutson et al. 2011). Our value for R_p is domi-

nated by the *Spitzer* $8 \mu\text{m}$ data due to the number of data points and high photometric precision. Since the magnitude of the aforementioned effects should be smallest at that wavelength range anyway, we calculate a global δ (Table 4) and do not let it vary for different data sets.

4.3. Planetary Phase Curve

In order to ascertain whether GJ 436’s thermal phase curve, i.e., the brightness variation as a function of orbital phase of the planet due to the longitudinal surface temperature distribution, could be characterized by observations, we calculate the predicted flux variation of GJ 436 b based upon the parameters shown in Tables 2 and 4. We follow the methodology outlined in Kane et al. (2011) and specifically Kane & Gelino (2011) that takes into account the eccentricity of the orbit. We assume a planet Bond albedo of zero and no stellar variation for our simulation purposes.

We perform the calculations for variable heat redistribution efficiencies (corresponding to η in Kane & Gelino 2011) within the planetary atmosphere between 0% and 100%. The resulting calculated flux ratio variations for the various models are shown in Figure 9.

In the case of 100% heat distribution efficiency (bottom right panel in Figure 9), the phase variation is only dependent on the changing star–planet distance due to the eccentric orbit, and the amplitude of the phase curve can be regarded as a lower limit. The eccentricity of the orbit results in a star–planet separation of 0.024 AU at periastron and 0.033 AU at apastron. For 100% heat redistribution efficiency, we calculate an equilibrium temperature of the planet of 718 K and 611 K at these locations in the orbit, respectively, following the formalism of Selsis et al. (2007). At a phase angle of zero, where secondary eclipse occurs, we calculate a star–planet separation of 0.027 AU and planetary equilibrium temperature of 675 K. The difference between the calculated equilibrium temperature and a value derived from observations during secondary eclipse for the planet day side will be indicative of the magnitude of brightness fluctuations on the planetary surface, potentially caused by the inefficiency of the heat redistribution. From their secondary eclipse observations, Deming et al. (2007) determine GJ 436’s dayside temperature to be 712 K, confirming that any observed phase curve would display larger amplitudes than our simulated one for perfect heat redistribution efficiency, making *Spitzer*

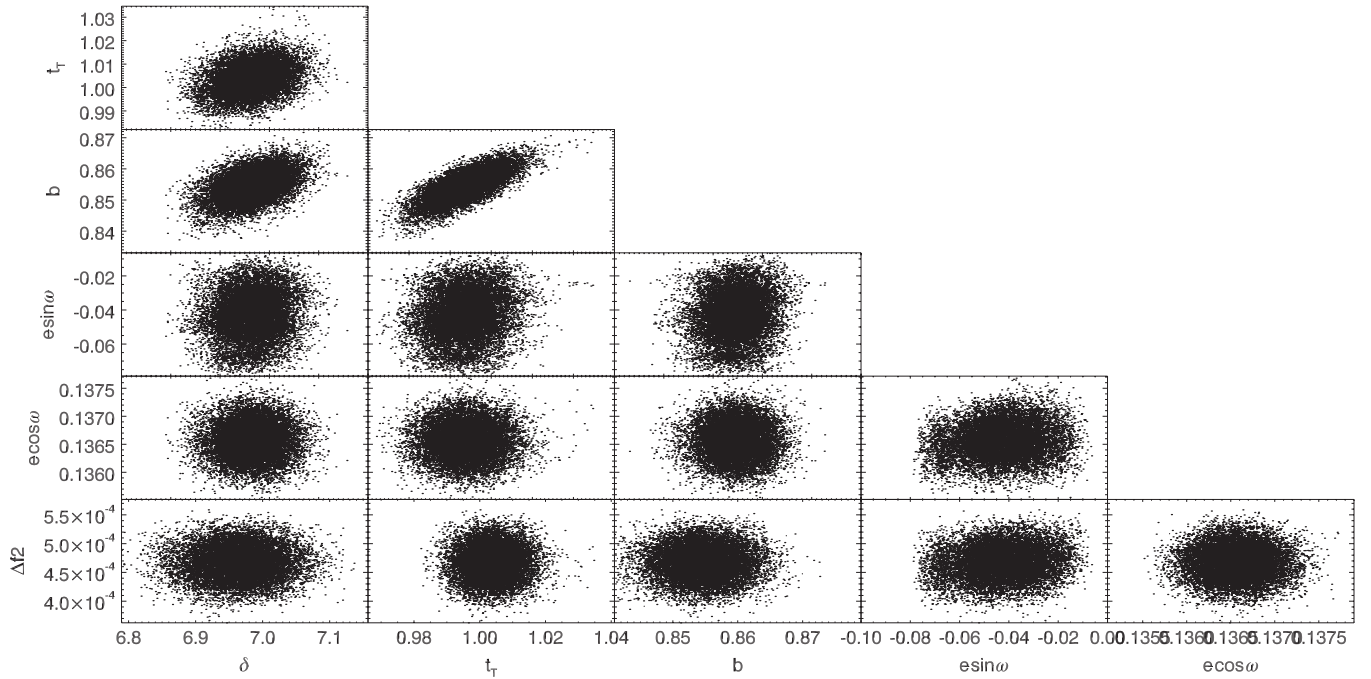


Figure 7. Correlations between some of our proposed parameters of the MCMC analysis. See Table 4 for explanation of symbols and units; δ , the transit depth, is shown here in mmag. For details, see Section 4.2.

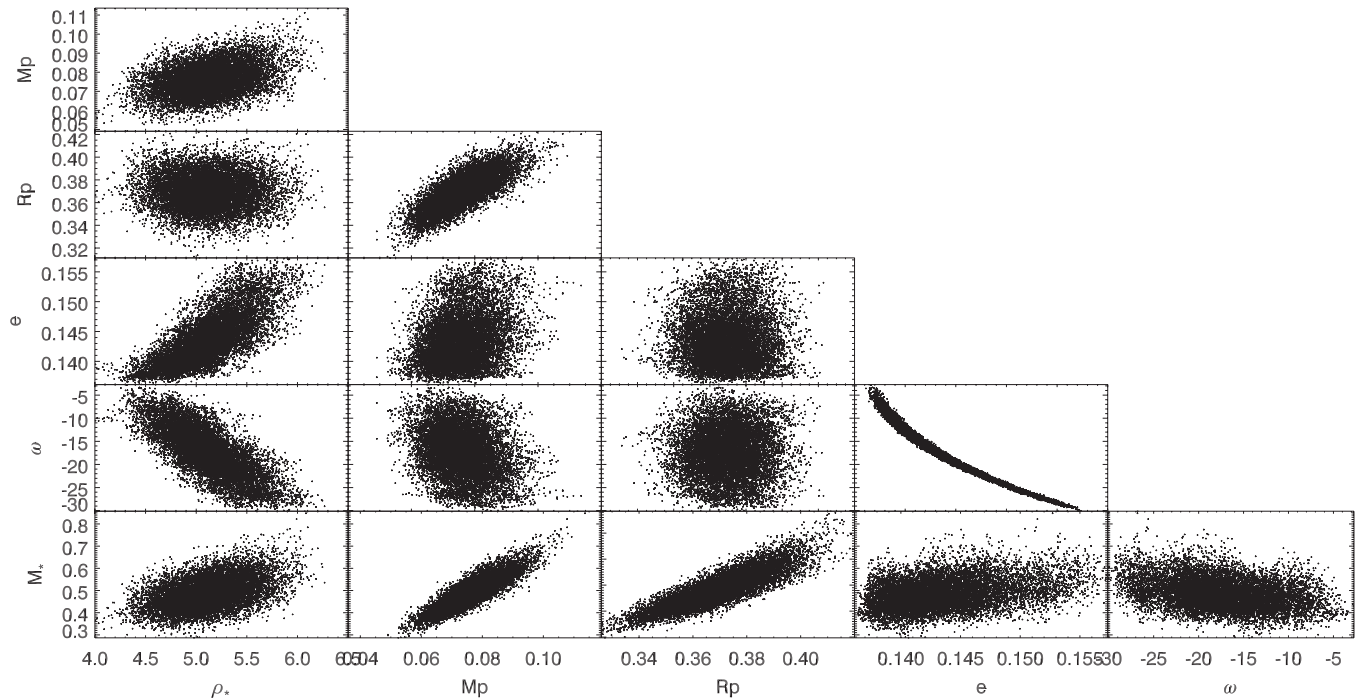


Figure 8. Correlations between some of the GJ 436 physical and orbital system parameters as calculated in our MCMC analysis. See Table 4 for explanation of symbols and units. For details, see Section 4.2.

follow-up observations of this target justified to place constraints on GJ 436's phase curve.

5. SUMMARY AND CONCLUSION

In this paper, we present a CHARA-Array interferometric radius for the transiting exoplanet, late-type host star GJ 436. Furthermore, we calculate a stellar effective temperature based solely on direct measurements. We present the values for these measurements in Table 2. We confirm the discrepancy between

stellar radii based on stellar models versus direct measurements (Boyajian et al. 2010) in the M dwarf regime (T. S. Boyajian et al. 2012, in preparation), which can also be seen in rapidly rotating, short-period eclipsing binaries (EBs) and active late type single stars (Torres & Ribas 2002; López-Morales & Shaw 2007; Hawley et al. 1996), as well as inactive field M dwarfs and long-period EBs (Boyajian et al. 2010; Irwin et al. 2011). Calculation of stellar radii based in part on the analysis of transit photometry, however, produces values in good agreement with our interferometric one.

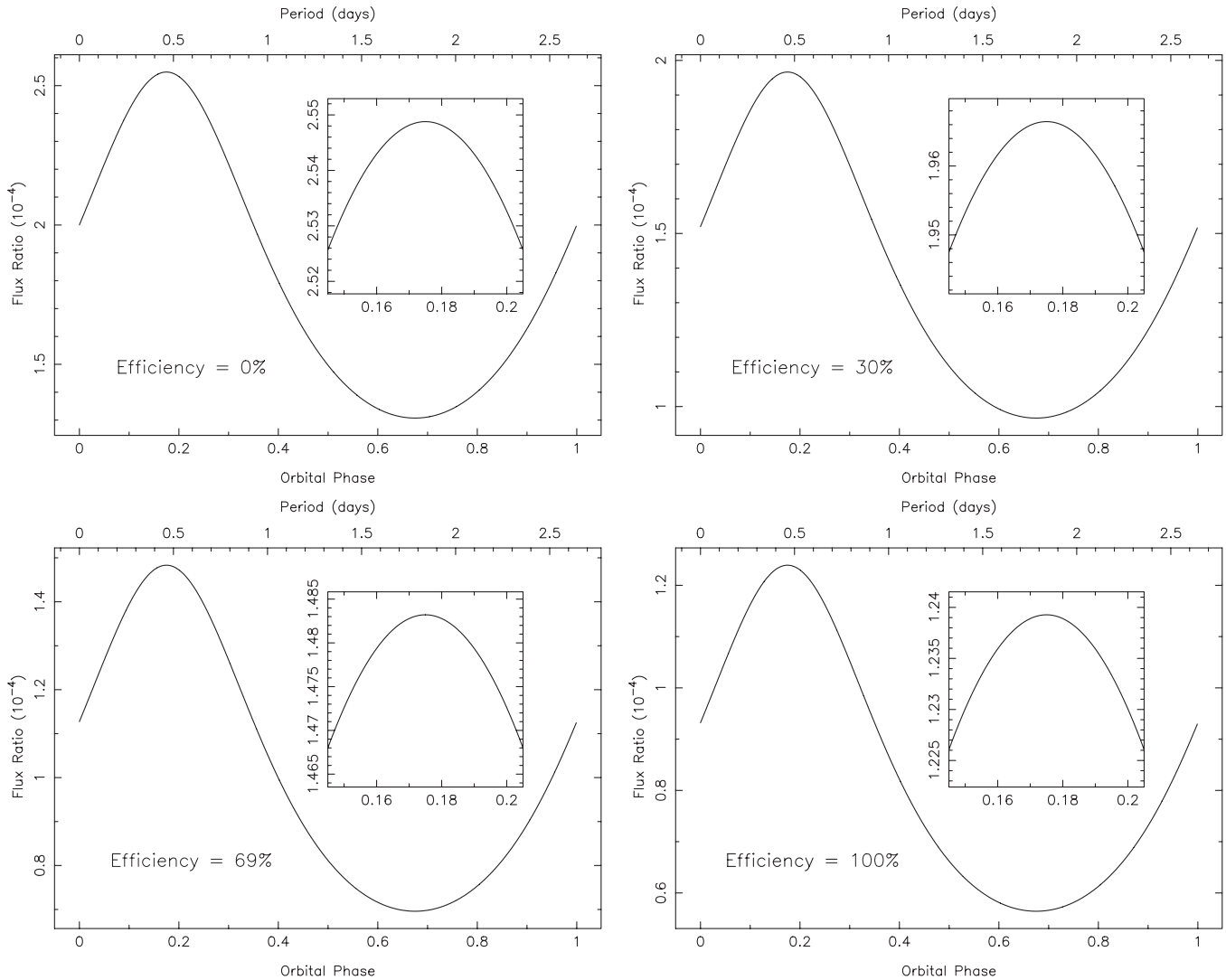


Figure 9. Simulated phase curve for GJ 436 at $4.5\ \mu\text{m}$, assuming 0%, 30%, 69%, and 100% heat redistribution efficiency of the atmosphere (equivalent to the η parameter in Kane & Gelino 2011). In each case, the sub-panel zooms in on the section of the phase curve around the peak in the planet-to-star flux ratio that would be optimal for monitoring. For details, Section 4.3.

We use our measured stellar properties in combination with literature time-series data available at the NASA Exoplanet Archive to perform a global analysis of the GJ 436 stellar and planetary system parameters, including radius and mass of the transiting hot Neptune. These calculated parameters are given in Table 4. Due to the aforementioned agreement between our measured radii and the ones obtained from transit photometry analysis, our calculated system parameters generally agree with literature values.

Planetary characterization is playing an increasingly dominant role in exoplanet research, especially of the nearby and/or bright stellar systems. The parent star obviously dominates the system as the principal energy source, and the object whose interaction with the exoplanet is often all that can be observed to characterize the planet as its own entity. Physical parameters of the planets are thus always a function of their stellar counterparts—the importance of “understanding the parent stars” cannot be overstated. With ongoing improvements in both sensitivity and spatial resolution of near-infrared and optical interferometric data quality, we are able to provide firm, direct measurements of stellar radii and effective temperatures in the

low-mass regime to provide a means of comparison to stellar parameters based on transit analysis.

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