CHARA TECHNICAL REPORT

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Polarization Revisited

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1. INTRODUCTION TO THE CHARA ARRAY PROJECT

The Center for High Angular Resolution Astronomy (CHARA) of Georgia State University will build a facility for optical/infrared multi-telescope interferometry, called the CHARA Array. This array will consist of initially five (with a goal of seven) telescopes distributed over an area approximately 350 m across. The light beams from the individual telescopes will be transported through evacuated pipes to a central laboratory, which will contain optical delay lines, beam combination optics, and detection systems. The facility will consist of these components plus the associated buildings and support equipment, and will be located at the Mount Wilson Observatory in southern California. The CHARA Array is funded by Georgia State University and the National Science Foundation.

2. OVERVIEW

The issue of polarization in interferometry has been discussed by several authors (Traub 1988, Beckers 1989, 1990), as well as in an earlier CHARA paper (Bagnuolo 1994). Heretofore, CHARA has adopted the strategy recommended by Traub. That is, all beam paths from all telescopes to the beam combination room are planned to follow an identical series of reflections and transmissions, in the same sequence (homologous beam paths), in order to ensure a similar state of polarization in all beams and no loss in source visibility.

The CHARA Array is now funded at a level which requires some simplifications in the planned facility. Some of these, related to optical path compensation, have been discussed in TR 4, where the use of fixed delay segments as a substitute for a full length variable delay, was described. Furthermore, with the selection of the CHARA site, the facility plan must also confront the reality of the site topography and existing facilities.

For these reasons, it is deemed necessary to reconsider the planned homologous beam paths. Specifically, we note that the introduction of additional, near-normal reflections may simplify the facility with minimal impact on polarization and visibility. In this report, the issue is examined more closely.

In the following, the discussion of phase difference at times refers to the difference between polarizations in a single wave, and at other times the difference between two waves.

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3. VISIBILITY LOSS DUE TO PHASE CHANGES ON REFLECTION

In general, a description of polarization effects in an interferometer will require a complete description of the polarization, for example as described in Bagnuolo (1994). This requires a complete description of the optics. However, some general conclusions can be drawn from a simpler description.

3.1. Non-polarized Light

Consider two plane, monochromatic waves E_1 and E_2, propagating parallel to the z axis, combined after traversing different optical paths. As a particular example, consider the case of unpolarized light (and ignore polarization which may be induced in the optical train itself), but allow for phase shifts which depend on the angle of the electric vector.

Describe the waves at combination by,

$$\vec{E}_1 = \vec{1}_x a(e^{i\delta_{1x}} + e^{i\delta_{1y}})e^{i\delta_1} \tag{1}$$

and

$$\vec{E}_2 = \vec{1}_y a(e^{i\delta_{2x}} + e^{i\delta_{2y}})e^{i\delta_2} \tag{2}$$

where $\delta_{1,2}$ represents a net phase shared by both vector component and $\delta_{1x,1y,2x,2y}$ represents phase shift particular to each vector component.

In a plane of superposition, the total electric intensity is

$$\vec{E}^2 = (\vec{E}_1 + \vec{E}_2)^2 \tag{3}$$
$$= \vec{E}_1^2 + \vec{E}_2^2 + 2\vec{E}1 \cdot \vec{E}2 \tag{4}$$
$$= 2a^2[1 + 0.5(\cos(\delta_x + \delta) + \cos(\delta_y + \delta)) \tag{5}$$

where $\delta_{x,y}$ are the phase differences in the two vector components and δ is the net phase difference between the two beams that is common to both vector components. With a trigonometric substitution and a shift of phase zero point, this can be written,

$$\vec{E}^2 = 2a^2[1 + \cos\frac{1}{2}(\delta_x - \delta_y)\cos\delta] \tag{6}$$

Setting $\delta_x = \delta_y$ recovers the familiar result,

$$E^2 = 2a^2(1 + \cos\delta). \tag{7}$$

So we see that a phase difference between vector components can only reduce the visibility, and the visibility transfer factor is given by

$$T_\phi = \cos\frac{1}{2}\ \delta_{xy}, \tag{8}$$

where δ_{xy} is the phase difference between the two vector directions.

In the extreme case, with a phase shift of π between the components, the visibility drops to zero. This case is easily achieved by including one extra reflection in one arm of an interferometer, resulting in a π phase change in one vector component and not in the other.

3.2. Polarized Light

As another example, consider the light which is polarized on transmission through the optics, but in this case ignore the direction-dependent phase shifts. For incident waves

$$E_1 = a_{1x}e^{i\gamma_1} + a_{1y}e^{i\gamma_1} \tag{9}$$

$$E_2 = a_{2x}e^{i\gamma_2} + a_{2y}e^{i\gamma_2} \tag{10}$$

and

$$E^2 = (a_{1x}a_{2x} + a_{1y}a_{2y})\cos\gamma \tag{11}$$

where γ is the net phase shift between waves.

In the unpolarized case, $a_{1x} = a_{1y} = a_{2x} = a_{2y} = a$, and the amplitude of modulation will be $2a^2$. In the Array optics train, both electric vectors will suffer partial absorption. If we assume that the absorption common to both vectors will be accounted elsewhere in estimating the efficiency, a differential absorption will remain. This will produce a partially polarized wave. From the definition of polarization (Born & Wolf, 1989; hereafter BW), the partial polarization of a wave, e.g., E_1, is

$$p = \frac{|a_{1x}^2 - a_{1y}^2|}{a_{1x}^2 + a_{1y}^2} \tag{12}$$

Suppose the differential absorption corresponds to an amplitude transmittance for one vector of t, or an intensity transmittance of $T = t^2$. Then we will have,

$$p = \frac{|1-T|}{1+T} \tag{13}$$

or,

$$t = \sqrt{\frac{1-p}{1+p}} \tag{14}$$

If both waves in the interferometer have a partial polarization p, the visibility transfer factor will have a value of

$$T_p = \frac{1}{1+p} \tag{15}$$

This is seen to have the correct behavior, giving a maximum apparent visibility transfer factor of 1.0 for $p = 0$ and a minimum of 0.5 for $p = 1$.

4. PHASE SHIFT AND POLARIZATION DUE TO REFLECTIONS

The primary source of phase shifts in the Array (other than the beam combiner itself) will be off-normal reflections from metallic mirrors. The phase shifts and polarization can be computed from the Fresnel formulae (BW).

For the case of reflection at the interface with a dielectric, the reflectivity for the electric vector parallel to the plane of incidence (p), or normal to the plane of incidence (n) may be written

$$r_p = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t} a_p \qquad (16)$$
$$r_n = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} a_s \qquad (17)$$

where $n_{1,2}$ are the indices of refraction in the ambient and reflective medium, respectively, θ_i is the angle of incidence, and θ_t is the angle of the transmitted component, which may be found from Snell's Law,

$$n_2 \sin\theta_t = n_1 \theta_i \qquad (18)$$

These expressions are also valid for reflection from conductive surfaces provided the appropriate complex value of the index of refraction $\hat{n}_2 = n_2(1 + i\kappa_2)$ is used. (This expression follows the notation of BW, but in more recent literature the quantity here denoted $n\kappa$ is often called k.)

The resulting complex reflection coefficient can be written,

$$r_p = \frac{n1 \cos\theta_i - (u_2 - iv_2)}{n1 \cos\theta_i + (u_2 - iv_2)} \qquad (19)$$
$$r_n = \frac{\left[n_2^2(1-\kappa_2^2)\cos\theta_i - n_1 u_2\right]^2 + \left[2n_2^2\kappa_2 \cos\theta_i - n_1 v_2\right]^2}{\left[n_2^2(1-\kappa_2^2)\cos\theta_i + n_1 u_2\right]^2 + \left[2n_2^2\kappa_2 \cos\theta_i + n_1 v_2\right]^2} \qquad (20)$$

where

$$2u_2^2 = n_2^2(1-\kappa_2^2) - n_1^2 \sin{\theta_i}^2 + \sqrt{[n_2^2(1-\kappa_2^2) - n_1^2 \sin{\theta_i}^2]^2 + 4n_2^4\kappa_2^2} \qquad (21)$$
$$2v_2^2 = -[n_2^2(1-\kappa_2^2) - n_1^2 \sin{\theta_i}^2] + \sqrt{[n_2^2(1-\kappa_2^2) - n_1^2 \sin{\theta_i}^2]^2 + 4n_2^4\kappa_2^2} \qquad (22)$$

The expressions for the amplitude factor and the phase shift of the reflected waves are also given by BW.

$$r_p^2 = \frac{(n_1 \cos\theta_i - u_2)^2 + v_2^2}{(n_1 \cos\theta_i + u_2)^2 + v_2^2} \qquad (23)$$
$$r_n^2 = \frac{\left[n_2^2(1-\kappa_2^2)\cos\theta_i - n_1 u_2\right]^2 + \left[2n_2^2\kappa_2 \cos\theta_i - n_1 v_2\right]^2}{\left[n_2^2(1-\kappa_2^2)\cos\theta_i + n_1 u_2\right]^2 + \left[2n_2^2\kappa_2 \cos\theta_i + n_1 v_2\right]^2} \qquad (24)$$

and

$$\tan\phi_p = \frac{2v_2 n_1 \cos\theta_i}{u_2^2 + v_2^2 - n_1^2 \cos{\theta_i}^2} \qquad (25)$$
$$\tan\phi_n = \frac{2n_1 n_2^2 \cos\theta_i 2\kappa_2 u_2 - (1-\kappa_2^2)v_2}{n_2^4(1+\kappa_2^2)^2 \cos{\theta_i}^2 - n_1^2(u_2^2+v_2^2)} \qquad (26)$$

5. OPTICAL CONSTANTS FOR SILVER

The CHARA optics will make extensive use of silver coated optics. The index of refraction for silver at several wavelengths is shown in Table 1. Values from three sources are shown, and as will be seen they are not entirely consistent. The sources are BW, HO (Handbook of Optics; Bass 1995), and CRC (CRC Handbook of Chemistry and Physics, 1985).

TABLE 1. Optical constants for silver.

Wavelength (μm)	n	$n\kappa$	Source
0.50	0.16	3.12	BW
0.50	0.24	3.09	CRC
0.56	0.12	3.45	HO
0.62	0.24	3.09	CRC
0.69	0.14	4.44	HO
0.75	0.18	3.9	BW
0.83	0.27	5.79	CRC
1.0	0.25	6.25	BW
1.03	0.23	6.99	HO
1.24	0.28	9.03	CRC
1.5	0.36	10.	BW
2.0	0.70	14.	BW
2.0	0.65	12.2	HO
2.47	0.67	18.32	CRC
2.5	1.0	16.	BW
3.10	0.91	22.89	CRC

From these constants, values for the reflectivity and differential phase shift $(p - n)$ are computed. Selected values are shown in Table 2 for the shortest wavelength considered in the Array design, $0.5\,\mu$m, for two sets of constants. The differential phase shifts are actually modulo 180°. The number of reflections in the beams will have to be adjusted to avoid a 180° phase difference.

The absorption and phase shifts are significantly smaller at longer wavelengths. Table 3 shows the computed values for two sets of constants near $2.0\,\mu$m. From these tables, we see that the predicted phase shift is relatively insensitive to the uncertainty of optical constants.

The predicted reflectivities differ significantly, and may all be wrong. Contemporary product information (e.g., Ferson Optics) claim reflectivities at normal incidence for protected silver coatings of 0.97 at $0.5\,\mu$m and 0.98 in the infrared.

Newport Corporation offers an overcoated silver coating called Broadband Infrared Reflector ER.2. The catalog shows reflectivity varying from 0.96 at $0.5\,\mu$m, increasing to 0.97, then decreasing to 0.96 at $0.9\,\mu$m, and increasing to 0.98 at $3\,\mu$m. Newport has provided us with computed reflectivity and phase data for a numerical model of their ER.2 coating. This computed information is not entirely consistent with the catalog reflectance, but is an interesting reference for such coatings.

Since this information is not published elsewhere, it is tabulated here for reference. Table 4 shows the reflectance and phase shift for the electric vector in the p and n planes. Data are shown for several angles of incidence, and over the wavelength range $0.5\,\mu$m to $2.4\,\mu$m. In order to better visualize these data, Figures 1 – 6 show the variation of reflectance and phase.

TABLE 2. Phase shift and reflectivity at 0.5 μm, according to BW and CRC.

Angle (deg)	BW Phase Diff. (deg)	BW r_p^2	BW r_n^2	CRC Phase Diff. (deg)	CRC r_p^2	CRC r_n^2
0.0	0.00000	0.94224	0.94224	0.00000	0.91340	0.91340
0.5	0.00280	0.94224	0.94224	0.00281	0.91341	0.91340
1.0	0.01115	0.94225	0.94223	0.01123	0.91342	0.91339
2.0	0.04464	0.94228	0.94220	0.04492	0.91346	0.91335
3.0	0.10047	0.94232	0.94215	0.10109	0.91353	0.91328
5.0	0.27922	0.94247	0.94200	0.28098	0.91375	0.91305
7.0	0.54775	0.94270	0.94178	0.55120	0.91408	0.91272
10.0	1.11990	0.94318	0.94129	1.12695	0.91479	0.91200
15.0	2.53150	0.94434	0.94010	2.54741	0.91650	0.91024
20.0	4.53108	0.94594	0.93839	4.55953	0.91887	0.90773
25.0	7.14552	0.94797	0.93616	7.19031	0.92188	0.90445
30.0	10.41486	0.95039	0.93337	10.48001	0.92547	0.90034
45.0	24.87502	0.95979	0.92121	25.02857	0.93945	0.88258
50.0	31.66475	0.96353	0.91583	31.85849	0.94503	0.87478
55.0	39.79747	0.96753	0.90989	40.03762	0.95100	0.86623
60.0	49.63592	0.97174	0.90373	49.92873	0.95731	0.85742

TABLE 3. Phase shift and reflectivity at 2.0 μm, according to BW and HO.

Angle (deg)	BW Phase Diff. (deg)	BW r_p^2	BW r_n^2	HO Phase Diff. (deg)	HO r_p^2	HO r_n^2
0.0	0.00000	0.98462	0.98462	0.00000	0.99310	0.99310
0.5	0.00054	0.98462	0.98461	0.00038	0.99310	0.99310
1.0	0.00217	0.98462	0.98461	0.00152	0.99310	0.99310
2.0	0.00870	0.98462	0.98461	0.00609	0.99311	0.99310
3.0	0.01957	0.98464	0.98459	0.01371	0.99311	0.99309
5.0	0.05440	0.98467	0.98456	0.03811	0.99313	0.99307
7.0	0.10675	0.98473	0.98450	0.07479	0.99315	0.99305
10.0	0.21843	0.98485	0.98438	0.15304	0.99321	0.99299
15.0	0.49470	0.98514	0.98408	0.34662	0.99334	0.99286
20.0	0.88790	0.98554	0.98363	0.62215	0.99352	0.99266
25.0	1.40541	0.98605	0.98304	0.98485	0.99375	0.99239
30.0	2.05829	0.98667	0.98225	1.44250	0.99402	0.99204
45.0	5.03668	0.98911	0.97833	3.53163	0.99512	0.99026
50.0	6.49896	0.99009	0.97622	4.55831	0.99556	0.98930
55.0	8.32102	0.99116	0.97344	5.83876	0.99604	0.98803
60.0	10.65565	0.99229	0.96968	7.48171	0.99655	0.98630

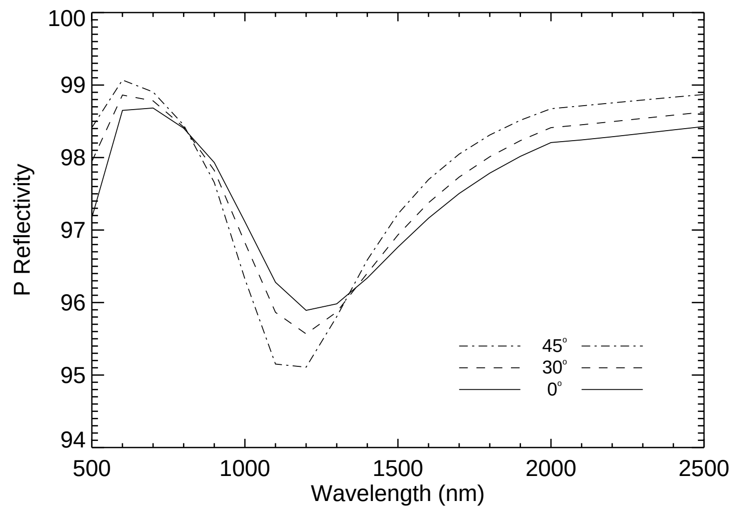

FIGURE 1. The computed n-plane reflectance for the Newport ER.2 silver coating.

Figures 1 and 2 show a substantial dip in reflectivity in the 1-1.5 μm range. This is certaily due to the ER.2 coating, perhaps a side effect of the an optimization in another wavelength range. Therefore this coating would not appear to be ideal for CHARA. However, as noted above, the Newport catalog graph of reflectivity does not show this dip.

Figures 3 and 4 show that the variation of phase with wavelength is much more important than the variation with angle of incidence. Figure 6 shows the phase shift change associated with small changes in angle of incidence (1 degree change). The changes are typically 1 degree of phase.

Comparing the computed phases for the ER.2 coating with those for the pure silver coating, the range of variation is similar, but the details differ considerably. In particular, the variation for small changes in angle around normal incidence are considerably larger for overcoated silver.

6. TOLERANCES

Differential phase shift effects produce a visibility transfer factor. This reduces the efficiency of the Array, and of course requires correction by calibration. Any variation of the visibility transfer will introduce additional demands on the calibration. The following comments are based on the worst case phase shifts, taken from the pure and overcoated silver coatings described above.

For illustration, assume that an acceptable visibility transfer factor will be 0.99. From Equation 8, the phase difference between two waves (ie two beams from different telescopes) must be less than 16°.

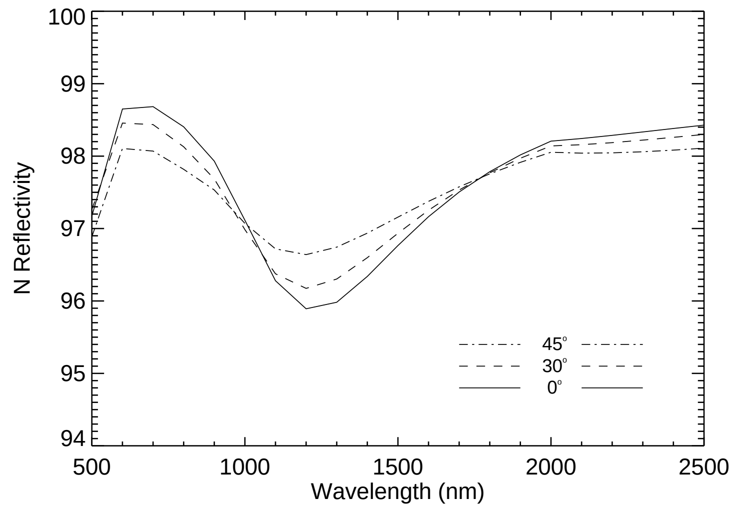

FIGURE 2. The computed p-plane reflectance for the Newport ER.2 silver coating.

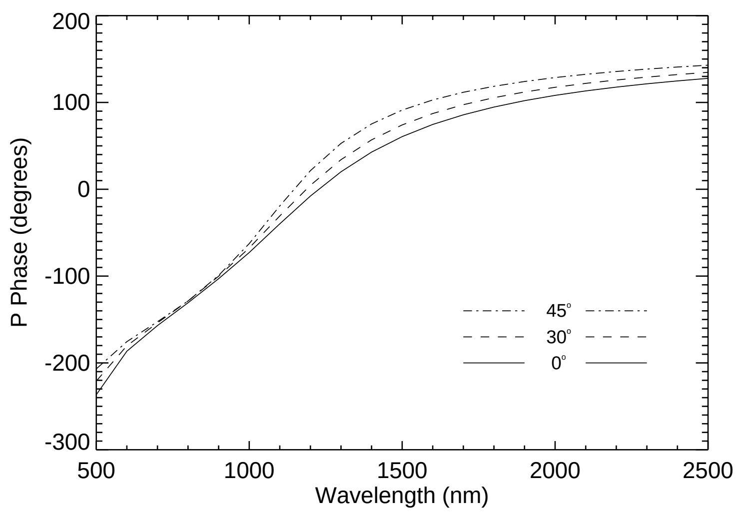

FIGURE 3. The computed n-plane phase shift for the Newport ER.2 silver coating.

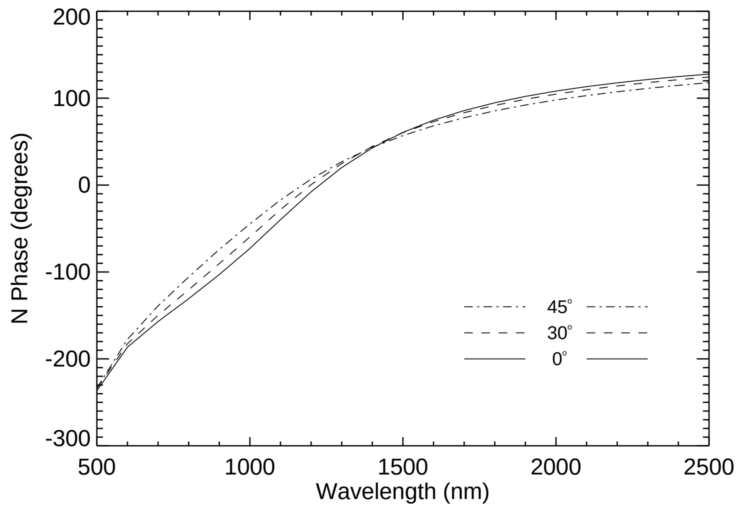

FIGURE 4. The computed p-plane phase shift for the Newport ER.2 silver coating.

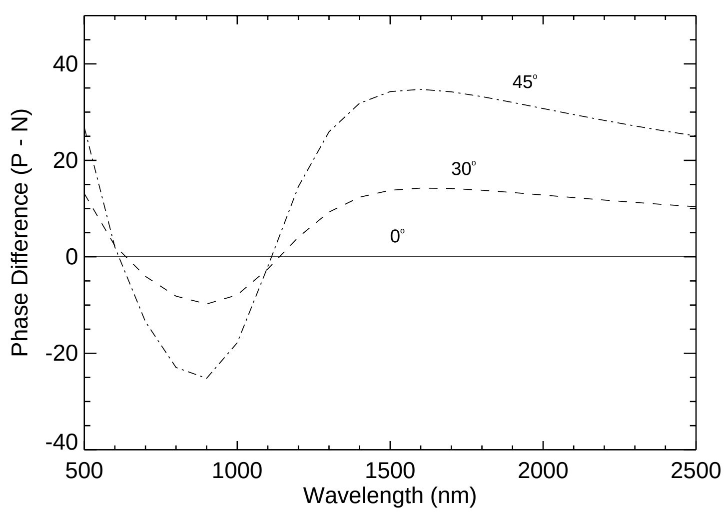

FIGURE 5. The computed phase difference between p-plane and n-plane phase shifts.

TABLE 4. Reflectance and phase shift in p and n planes.

Angle of Incidence (deg)	Wavelength (nm)	*P*-Plane Reflectivity (%)	*P*-Plane Phase (deg)	*N*-Plane Reflectivity (%)	*N*-Plane Phase (deg)
0	500	97.171	−236.588	97.171	−236.588
0	600	98.650	−186.496	98.650	−186.496
0	700	98.683	−156.981	98.683	−156.981
0	800	98.405	−130.769	98.405	−130.769
0	900	97.931	−102.936	97.931	−102.936
0	1000	97.115	−72.842	97.115	−72.842
0	1100	96.279	−39.785	96.279	−39.785
0	1200	95.892	−7.778	95.892	−7.778
0	1300	95.982	20.145	95.982	20.145
0	1400	96.339	42.824	96.339	42.824
0	1500	96.766	60.674	96.766	60.674
0	1600	97.164	74.675	97.164	74.675
0	1700	97.504	85.775	97.504	85.775
0	1800	97.785	94.716	97.785	94.716
0	1900	98.016	102.044	98.016	102.044
0	2000	98.207	108.148	98.207	108.148
0	2100	98.243	113.272	98.243	113.272
0	2200	98.287	117.667	98.287	117.667
0	2300	98.334	121.478	98.334	121.478
0	2400	98.381	124.818	98.381	124.818
0	2500	98.428	127.770	98.428	127.770
30	500	97.946	−220.858	97.263	−233.969
30	600	98.862	−180.312	98.456	−182.607
30	700	98.782	−153.862	98.435	−149.804
30	800	98.421	−128.653	98.129	−120.493
30	900	97.823	−100.112	97.685	−90.327
30	1000	96.826	−67.534	96.985	−59.629
30	1100	95.865	−30.703	96.375	−28.182
30	1200	95.568	4.581	96.173	0.504
30	1300	95.870	34.103	96.303	24.819
30	1400	96.405	56.922	96.598	44.562
30	1500	96.933	74.151	96.934	60.353
30	1600	97.377	87.258	97.253	73.010
30	1700	97.731	97.430	97.532	83.267
30	1800	98.011	105.504	97.770	91.695
30	1900	98.234	112.054	97.971	98.723
30	2000	98.413	117.470	98.141	104.663
30	2100	98.452	121.992	98.158	109.711
30	2200	98.496	125.856	98.186	114.085
30	2300	98.540	129.197	98.221	117.915
30	2400	98.585	132.118	98.259	121.296
30	2500	98.628	134.695	98.299	124.306

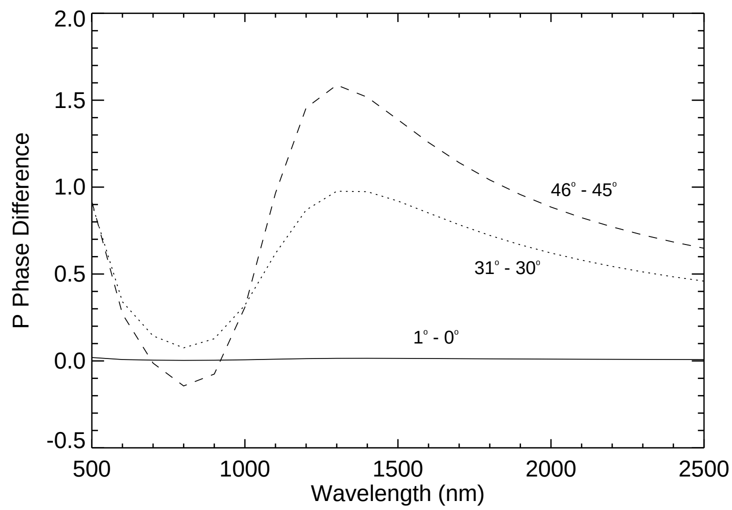

FIGURE 6. The computed p-plane phase change for a change in the angle of incidence by 1°.

TABLE 4. (continued).

Angle of Incidence (deg)	Wavelength (nm)	P-Plane Reflectivity (%)	P-Plane Phase (deg)	N-Plane Reflectivity (%)	N-Plane Phase (deg)
45	500	98.413	−206.822	96.887	−233.669
45	600	99.070	−175.460	98.105	−177.472
45	700	98.905	−152.536	98.070	−139.063
45	800	98.449	−128.720	97.818	−105.778
45	900	97.656	−99.223	97.532	−74.042
45	1000	96.325	−62.547	97.071	−44.725
45	1100	95.151	−19.100	96.719	−17.074
45	1200	95.108	21.445	96.640	6.876
45	1300	95.805	52.837	96.743	26.915
45	1400	96.588	75.257	96.936	43.414
45	1500	97.222	91.218	97.158	56.970
45	1600	97.696	102.895	97.375	68.172
45	1700	98.047	111.728	97.576	77.521
45	1800	98.311	118.622	97.756	85.409
45	1900	98.514	124.151	97.914	92.140
45	2000	98.674	128.686	98.053	97.944
45	2100	98.713	132.452	98.041	102.959
45	2200	98.754	135.655	98.045	107.370
45	2300	98.794	138.418	98.060	111.280
45	2400	98.833	140.827	98.083	114.772
45	2500	98.870	142.949	98.110	117.909

6.1. Alignment Tolerances

The specification of homologous optical systems is based on the expectation that identical phase shifts will be introduced in all beam paths. In practice, alignment errors will partially negate this assumption.

For a reflection at an angle of incidence of 45°, an error in the angle of incidence will result in a change in the phase difference of no more than 1.°5 per degree of error. Assuming that errors combine quadratically, the net phase error for n reflections, with a mean error of e degrees will be,

$$E = 1.5\, e\sqrt{n} \tag{27}$$

For a typical value of $n = 15$, and a maximum cumulative phase error of 16°, the mean alignment error e must be on the order of 3° or less.

6.2. Uncompensated Reflections

From Table 2, it is evident that for pure silver a number of uncompensated reflections at small angle of incidence (no greater than a few degrees) could be tolerated. However, for the ER.2 coating, a single uncompensated reflection will introduce a significant and complex phase error (Figures 3 and 4). If this is generally true of protected silver coatings, it may be necessary to employ pure silver for any such reflections.

6.3. Variation of Visibility Transfer Factor

The telescopes contain reflections which will partially polarize the beams, with a resulting reduction in visibility. This polarization will be dependent on the telescope orientation.

The Array optics will introduce partial polarization, which will ideally be harmless as long as it is constant and common to all beams. However, pointing of the telescopes around the sky will result in varying polarization of the beams, which must be accounted for in calibration. Pending a more elaborate analysis, an estimate can be made of the magnitude of the effect.

Considering first the case of silver coatings, a single 45° reflection at $0.5\,\mu$m will introduce a partial polarization of about 3%. As the telescope rotates in altitude only, the partial polarization will rotate in angle, with a resulting variation of the partial polarization amplitude of each vector component of about 6%. The variation with zenith distance ζ can be described by,

$$p(\zeta) = 0.03[3 + \cos(2\zeta)] \tag{28}$$

The visibility transfer factor due to this polarization will be (Eq. 15),

$$T_p(\zeta) = \frac{1}{1+p} = \frac{1}{1 + 0.03[3 + \cos(2\zeta)]} \tag{29}$$

If the visibility is required to change by no more than 0.5% during an offset from source to calibration star, the offset in zenith distance at 45° must be no more than about 5°.

A similar analysis applies for the case of telescope rotation in azimuth only, and of course for motions in both altitude and azimuth, variations in polarization of amplitude 12% are possible. This is an important variation, which can probably be accommodated by choosing calibration sources carefully and computing corrections for the residual change in the visibility transfer factor.

The ER.2 coatings produces similar amplitude of polarization, though with a different wavelength dependence. It will also be important to understand whether the uniformity of the coatings is sufficiently high to justify the plan of compensating polarization phase shifts with homologous optics.

7. CONCLUSIONS

Further investigation of available protected silver coatings is needed. Computed characteristics should be used only as a guide and measured characteristics should be sought for confirmation.

8. REFERENCES

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